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A Study on Optimization of Fading Memory Polynomial Filter to Track High Mobility Warship
本論文之 潘寶峰 工學博士 學位論文 認准為

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고 기동성 군함 추적용 페이딩 메모리 다항식 필터의 최적화 연구

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초록

추적 필터는 선박의 위치와 속도를 정확하게 예측하는 데 중요한 역할을 한다. 추적에는 여러 가지 방법이 사용된다. 그러나 가장 일반적으로 사용되는 방법은 칼만 필터와 그것에서 파생되는 방법이다. $\alpha-\beta-\gamma$ 필터는 칼만 필터가 제공하는 일반적인 솔루션의 특별한 경우 중 하나이다. $\alpha-\beta-\gamma$ 필터의 많은 알고리즘 중에서 페이딩 메모리 다항식 필터(fading memory polynomial filter, FMP filter)는 정확도가 더 높으며 쉽게 실행이 가능하다.

군함은 주로 해상 전쟁을 목적으로 제작된 해군 함선을 일컫는다. 무기를 장착함은 물론이고 충격을 견디 수 있도록 고안되었으며 일반적인 상선보다 속도가 더욱 빠르고 기동성도 좋다. 군함은 감착스럽게 속도나 항로를 변경할 때 뛰어난 기동성을 발휘하므로 추적하기가 어려워진다. 한편 추적 정확도는 해상상황과 기상조건에 따라 좌우되기도 한다. 따라서 기동성이 좋은 군함을 추적하는 것은 항해에 있어 중요한 것으로 인식되고 있다.

본 연구에서는 일반적으로 사용되는 필터 알고리즘과 비교하여 추적 정확도가 높고 계산량이 적은 FMP 필터가 다른 필터에 비하여 우수함을 제시하였다. 최적화 된 3차 FMP 필터는 본선이 고정되어 있거나 움직인
때도 높은 기동성 목표물을 잘 추적할 수 있다. 그러나 군함의 속도가 매우 빠르고 침로의 변화가 심하기 때문에 3차 FMP 필터는 추적 정확도가 떨어진다. 따라서 본 연구에서는 가속도의 변화를 수정할 수 있는 4차 FMP 필터로 확장하였다.

속도변화에 따라 최적화 된 4차 FMP 필터를 이 논문에서 제시하였다. 이 새로운 필터를 사용하면 추적 정확도를 더욱 향상시킬 수 있다. 동시에 속도에 따른 최적한 파라미터 $\xi$ 를 이용하면 추적하기 전에 필터 계수 찾은 시간도 절약 할 수 있다. 3000번 실험 결과를 통계하고 랜덤으로 생산되는 궤적 추적용 4차 FMP 필터의 파라미터 $\xi$ 값의 범위가 [0.44, 0.6] 으로 확인되었다.

추적 정확도는 추적 필터의 성능을 평가하는 요소 중 가장 중요한 요소이다. 추적 필터의 정확도가 항상되면 실제 상황에서 더욱 정확한 위치를 차지해 전쟁의 주도권을 잡을 수 있을 것이다. 실제 응용적 측면에서 글린트 노이즈, 다수의 목표물의 추적 등은 어떻게 해결할 것이다와 레이더 영상은 방위와 거리로 주어지는 바 이것에 대한 오차는 어떻게 처리할 것인지 추후 연구로 남겨둔다.

KEY WORDS 추적; 정확도; 페이딩 메모리 다항식 필터; 갈만 필터; 3차 필터; $\alpha-\beta-\gamma$ 필터; 4차 필터; $\alpha-\beta-\gamma-\eta$ 필터.
A Study on Optimization of fading memory polynomial filter to Track High Mobility Warship

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Abstract

The tracking filter plays a key role in accurate estimation and prediction of maneuvering vessel's position and velocity. Different methods are used for tracking. However, the most commonly used method is the Kalman filter and its modifications. The $\alpha-\beta-\gamma$ filter is one of the special cases of the general solution provided by the Kalman filter. In numerous of algorithms of $\alpha-\beta-\gamma$ filter, the fading memory polynomial filter (FMP filter) performs better accuracy and is easy to implement.

A warship is a naval ship that is built and primarily intended for naval warfare. As well as being armed, warships are designed to withstand damages and are usually faster and more maneuverable than merchant ships. As warship has excellent performance in sudden changes of speed and course, it brings difficulty in tracking. Meanwhile the sea and weather conditions also affect the tracking accuracy. Tracking a high mobility warship is a widely recognized challenge in navigation.

This study compares the commonly used filter algorithms and gives out the superiority of FMP filter, which has the higher tracking accuracy
and less computations. After optimization, the third-order FMP filter performs good in tracking high mobility target under the conditions of own ship is in motion or motionless. However, the dynamics of warship changes very fast and frequently, and the third-order FMP filter is powerless to track. Therefore, it is extended to the fourth-order FMP filter that adds the correction of jerky which stands for the differential of acceleration.

Speed-dependent fourth-order FMP filter is also illustrated in this paper. With this new tracking method, the tracking accuracy can be further improved. Meanwhile, the usage of speed gain function can also save time to find appropriate parameters. The statistical results of 3000 times of simulation show that the recommended interval [0.44, 0.6] is given to track trajectories which were randomly generated.

Tracking accuracy is the most important factor to evaluate the performance of tracking filter. The improvement of accuracy of tracking filter can obtain much more accurate position and take the lead in war. The future study will deal with the problems such as controlling of glint noise and tracking multiple objects in practical applications. Meanwhile the error which calculated by the range and bearing presented on the radar system will also be considered in future study.

**KEY WORDS:** Tracking; Accuracy; Fading Memory Polynomial Filter; FMP Filter; Kalman Filter; Third-order Filter; $\alpha-\beta-\gamma$ Filter; Fourth-order FMP Filter.
Chapter 1 Introduction

1.1 Background and Purpose of This Study

With the progress and development of science and technology, tracking is widely used in self-driving in automobile industry, ship automatic navigation, satellite positioning and other fields. In these applications, they have a common feature that the tracking is applied to get the accurate position of target. Tracking filters are serial algorithms which occupy the core position in tracking system.

In the military field tracking filter is widely used in precision guidance system, radar positioning system, fighter positioning system and warship positioning system. During the war or for the purpose of performance improvement, the positioning system plays a vital and decisive role. The observed position of target, which includes internal and external noise errors, can be obtained by radar systems. In order to obtain much more accuracy position of target, tracking filter is used to reduce noise. With the data of predicted position, which is the output data of filter, the commanders can take their next step.

A warship has excellent performance in sudden changes of speed and course, it is difficult to track. Meanwhile the sea and weather conditions also affect the tracking accuracy. Tracking for the high mobility warship, therefore, is a widely recognized challenge in navigation.

This study aims to use theoretical simulations method to optimize the fading memory algorithm, which performs the best after comparing with other common filters. Speed-dependent self adaptive filter algorithms are developed to track high mobility warship. Compared with the constant
coefficient filter, speed-dependent filter has advantages of high accuracy and saving time of finding parameters.

1.2 Warship and Its Properties

A warship is a naval ship that is built and primarily intended for naval warfare. Usually they belong to the armed forces of a state. As well as being armed, warships are designed to withstand damage and are usually faster and more maneuverable than merchant ships. Unlike a merchant ship, which carries cargo, a warship typically carries only weapons, ammunition and supplies for its crew. Warships usually belong to a navy, though they have also been operated by individuals, cooperatives and corporations.

Table 1.1 Statistics of speed of warships commissioned in past 30 years

<table>
<thead>
<tr>
<th>Category</th>
<th>Name</th>
<th>Builders</th>
<th>Displacement Tons</th>
<th>Commissioned Year</th>
<th>Speed Knot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aircraft carriers</td>
<td>Admiral Kuznetsov-class (Project 1143.5) aircraft carrier</td>
<td>Soviet Union</td>
<td>67,000</td>
<td>1995</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>Cavour-class aircraft carrier</td>
<td>Italy</td>
<td>27,910</td>
<td>2008</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Chakri Naruebet-class aircraft carrier</td>
<td>Spain</td>
<td>11,486</td>
<td>1997</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>Charles de Gaulle-class nuclear-powered aircraft carrier</td>
<td>France</td>
<td>40,500</td>
<td>2001</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Gerald R. Ford-class nuclear-powered aircraft carrier</td>
<td>United States</td>
<td>100,000</td>
<td>2017</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Vikramaditya (modified Kiev)-class aircraft carrier</td>
<td>India/Russia</td>
<td>45,400</td>
<td>2013</td>
<td>30</td>
</tr>
<tr>
<td>Destroyers</td>
<td>052 type destroyer</td>
<td>China</td>
<td>4,800</td>
<td>1993</td>
<td>30</td>
</tr>
<tr>
<td>(NATO codename Luhu)</td>
<td>051B type destroyer (NATO codename Luhai)</td>
<td>China</td>
<td>6,100</td>
<td>1999</td>
<td>30</td>
</tr>
<tr>
<td>---------------------</td>
<td>------------------------------------------</td>
<td>-------</td>
<td>-------</td>
<td>------</td>
<td>----</td>
</tr>
<tr>
<td>052B type destroyer (NATO codename Luyang I)</td>
<td>China</td>
<td>5,850</td>
<td>2004</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>051C type destroyer (NATO codename Luzhou)</td>
<td>China</td>
<td>7,100</td>
<td>2006</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>052C type destroyer (NATO codename Luyang II)</td>
<td>China</td>
<td>6,500</td>
<td>2004</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>052D type destroyer (NATO codename Luyang III)</td>
<td>China</td>
<td>7,500</td>
<td>2014</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Arleigh Burke-class destroyer</td>
<td>United States</td>
<td>9,200</td>
<td>1991</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Atago-class destroyer</td>
<td>Japan</td>
<td>10,000</td>
<td>2007</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Chungmugong Yi Sun-sin (KDX-II)-class destroyer</td>
<td>South Korea</td>
<td>5,500</td>
<td>2003</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Delhi-class destroyer</td>
<td>India</td>
<td>6,700</td>
<td>1997</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>Gwanggaeto the Great (KDX-1 Okpo)-class destroyer</td>
<td>South Korea</td>
<td>3,900</td>
<td>1998</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Horizon-class destroyer</td>
<td>France/Italy</td>
<td>5,600</td>
<td>2007</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Kolkata-class destroyer</td>
<td>India</td>
<td>7,500</td>
<td>2014</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Kongo-class destroyer</td>
<td>Japan</td>
<td>9,485</td>
<td>1993</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Murasame-class destroyer</td>
<td>Japan</td>
<td>4,550</td>
<td>1996</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Sejong the Great (KDX-III)-class destroyer</td>
<td>South Korea</td>
<td>10,290</td>
<td>2007</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Type 45 destroyer</td>
<td>United Kingdom</td>
<td>7,205</td>
<td>2009</td>
<td>29</td>
<td></td>
</tr>
<tr>
<td>Zumwalt-class destroyer</td>
<td>United States</td>
<td>14,564</td>
<td>2016</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Type</td>
<td>Name</td>
<td>Country</td>
<td>Displacement</td>
<td>Year</td>
<td>Delivery</td>
</tr>
<tr>
<td>------------------------------</td>
<td>-------------------------------------------</td>
<td>------------</td>
<td>--------------</td>
<td>------</td>
<td>----------</td>
</tr>
<tr>
<td>Frigates</td>
<td>053H2G type frigate (NATO codename Jiangwei I)</td>
<td>China</td>
<td>2,250</td>
<td>1992</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>054A type frigate (NATO codename Jiangkai II)</td>
<td>China</td>
<td>3,600</td>
<td>2008</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Abukuma-class destroyer escort</td>
<td>Japan</td>
<td>2,550</td>
<td>1989</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>Álvaro de Bazán (F100)-class frigate</td>
<td>Spain</td>
<td>6,250</td>
<td>2002</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>Fridtjof Nansen-class frigate</td>
<td>Spain</td>
<td>5,121</td>
<td>2006</td>
<td>26</td>
</tr>
<tr>
<td></td>
<td>F-22P Zulfiqar-class frigate</td>
<td>Pakistan/China</td>
<td>3,144</td>
<td>2009</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>La Fayette-class frigate</td>
<td>France</td>
<td>3,280</td>
<td>1995</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Shivalik-class frigate</td>
<td>India</td>
<td>6,200</td>
<td>2010</td>
<td>32</td>
</tr>
<tr>
<td>Corvettes</td>
<td>056 type corvette (NATO codename Jiangdao)</td>
<td>China</td>
<td>1,300</td>
<td>2013</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Durjoy-class corvette</td>
<td>China</td>
<td>650</td>
<td>2012</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>Khukri-class corvette</td>
<td>India</td>
<td>1,350</td>
<td>1990</td>
<td>25</td>
</tr>
<tr>
<td></td>
<td>Milgem-class corvette</td>
<td>Turkey</td>
<td>2,400</td>
<td>2011</td>
<td>30</td>
</tr>
<tr>
<td>Large patrol vessels</td>
<td>River-class patrol vessel</td>
<td>United Kingdom</td>
<td>1,677</td>
<td>2003</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>Samuel Beckett / P60-class offshore patrol boat</td>
<td>United Kingdom</td>
<td>1,933</td>
<td>2014</td>
<td>20</td>
</tr>
<tr>
<td>Minor Surface combatants</td>
<td>Ambassador Mk III fast attack craft</td>
<td>United States</td>
<td>500</td>
<td>2013</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>Skjold-class patrol boat</td>
<td>Norway</td>
<td>274</td>
<td>1999</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>Houjian (Type 37-II)-class large missile boat</td>
<td>China</td>
<td>542</td>
<td>1991</td>
<td>33.5</td>
</tr>
<tr>
<td>Air Cushioned Landing Crafts</td>
<td>Landing Craft Air Cushion (LCAC 91) Air Cushioned Landing Craft</td>
<td>United States</td>
<td>87.2</td>
<td>2001</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>Zubr class (Project)</td>
<td>Soviet</td>
<td>555</td>
<td>1988</td>
<td>63</td>
</tr>
</tbody>
</table>
Table 1.1 is the speed survey result of warships which were commissioned in past 30 years. The vast majority of warships have an upper limit speed of 30 knots. Some warships which were built by Soviet Union can reach to 32 knots. As some Indian warships were imported from Soviet Union, most of Indian warships also have relatively high upper limit speed of 32 knots. Minor Surface combatants and air cushioned landing crafts have fairly high speed of 70 knots, but the maneuverability is relatively lower than other types of warship.

In order to be able to generally reflect the speed level of the warships, in particular the relative speed of the relative motions, an initial value of 50 \( m/s \) is adopted in this paper as the initial value of speed of simulation.

1.3 Introduction of Fading Memory Polynomial Filter (FMP Filter)

Kalman filter is well-known and widely used in tracking field. But other filters exist and they perform better than Kalman filter. The FMP filter is recursive and the weights of new measurements are more heavily than older measurements. Its structure is nearly identical to the linear polynomial Kalman filter, but its algorithm for gain calculation is much simpler than solving Riccati equation, and the FMP filter is therefore less computationally burdensome. FMP filter is quite popular in radar tracking applications and can also be competitive with Kalman filters in radar tracking applications.
1.4 Related Studies of the Study

Due to the essential role that tracking filters play in a tracking system, many researchers have taken quite an interest in understanding the theory and application leading to valuable insights into design developments and improvement. In the early work of Benedict and Bordner (1962), the dual-based their analysis of the $\alpha-\beta$ filter stays in the frequency domain ($Z$-transform). They proposed a relationship between the $\alpha$ and $\beta$ filtering coefficients derived from a pole matching technique in order to optimize the tracker’s ability to reduce noise and achieve a good transient performance. This led to what is known today as the Benedict-Bordner relationship. Simpson (1963) further extended this study to the $\alpha-\beta-\gamma$ filter by including the acceleration term, and thus arriving at the optimization condition between the filter weight coefficients.

In his study ‘A generalized parameter for $\alpha-\beta$ and $\alpha-\beta-\gamma$ target trackers’, Kalata (1984) proposes the use of a tracking index that relates the filter coefficients and is a function of position uncertainty due to target maneuverability, radar measurement uncertainty and update time interval. He utilizes the tracking index parameter to derive implicit closed form equations of the smoothing coefficients which results in optimal performance. A more convenient way to determine the optimal filtering weights was investigated by Gray and Murray (1993) whereby a damping parameter that computes the smoothing coefficients directly was derived analytically.

In the recent past other researchers have been involved further in the investigation of the optimal $\alpha-\beta-\gamma$ filter algorithm aimed at striking a balance between good noise reduction and transient response capability.
Tenne and Singh (2002) derived closed form solutions of an optimal \( \alpha - \beta - \gamma \) filter whose performance was based on noise reduction ratio, steady state maneuver error and transient response for circular and straight line trajectories and subsequently determining a figure of demerit. Blair (1991) introduces a two-stage \( \alpha - \beta - \gamma \) filter whose second stage is augmented by incorporating the kinematic constraint of constant speed.

In this study, different algorithms of the steady state Kalman filter are investigated and compared based on capability to reduce noise of a maneuvering target and hence provide quality estimates. The study focuses on performance comparison of the Benedict-Bordner filter also known as the Simpson filter, Gray & Murray filter and the third-order FMP filter. The FMP filter was optimized by adjusting the value of the damping parameter, \( \xi \), experimentally as discussed further by Pan (2016) where it was explicitly noted that the optimum set of the smoothing coefficients depends on the average speed of the target under consideration.

Considered the tracking accuracy, the \( \alpha - \beta \) and \( \alpha - \beta - \gamma \) filters are limited in their capacity to follow a high maneuvering target, defined by a jerky motion, with good accuracy. The jerky motion is reduced considerably when the target tracking equations are modelled to make provision for a constant jerk. The third order \( \alpha - \beta - \gamma \) filter, therefore, extends to a fourth-order filter when the design is extended to include the constant jerk parameter.

Several approaches have been introduced recently in an attempt to design filtering equations that model for the constant jerk. They have been found to out-perform the constant acceleration \( \alpha - \beta - \gamma \) filter in terms of error reduction and ability to follow a maneuvering target with jerky movements. Mehrotra (1997) suggested a jerk model for tracking highly
maneuvering targets marred by unexpected changes in the speed. The simulation results indicate better performance of the jerk model than that of the lower order filters. The study undertaken by Chen (2008) also showed an improvement in the target tracking accuracy when the \( \alpha-\beta-\gamma-\sigma \) filter was utilized in tracking a constant jerk model compared to the conventional \( \alpha-\beta-\gamma \) filter.

Wu (2011) went further and proposed an evolutionary programming based on \( \alpha-\beta-\gamma-\sigma \) filter that provided an optimal simulation technique for a maneuvering target with jerky movement. In addition, with highly accurate and efficient prediction of the target trajectory, this new fourth-order filter was associated with a reduced computational time. In Wu’s research, estimation factors were given by corresponding intervals and the parameters were determined by Taguchi method, which was complex and needed lots of computations. Incidentally, the mathematic model of estimation has a mistake in Equation 6 and 7 in his paper, the numbers 2 and 6 should be contained in the denominator.

As warship has excellent performance in sudden changes of speed and course, traditional filter models are difficult to track. Therefore, this paper optimizes third-order FMP filter and fourth-order FMP filter to improve the tracking accuracy for tracking high mobility target. Meanwhile, speed-dependent self-adaptive filter is also developed to further improve tracking accuracy and save time.

1.5 Scope, Methodology and Content of This Study

The scope of this paper is to optimize the fading memory algorithms of third order FMP filter and fourth order FMP filter by MATLAB simulation.

In each simulation, the original positions are generated by trigonometric
functions firstly. Then sampling it by 3 seconds, which is the same frequency of radar, to get the true position. After adding error into the X-axis and Y-axis, the observation can be obtained as the input data of filter. Two important factors of output of filter, predicted position and smoothed position, are used to measure the performance of filter. Finally, by changing the original position generation model to vary the speed of input model, a $\xi$-speed curve can be fitted.

As the content of this study shown in Chapter 2, the fading memory algorithms and their advantages are described. Chapter 3 analyzes the third order fading memory algorithms meanwhile Chapter 4 analyzes the fourth order fading memory algorithms and compares the performance of two filters. Finally, conclusions of this study are stated in Chapter 5.
Chapter 2 FMP Filter and Its Superiority

This Chapter introduces the algorithms of the third-order and fourth-order FMP filter. After comparing with commonly used $\alpha-\beta-\gamma$ filter algorithms and Kalman filter algorithm, the superiority of FMP filter will be summarized.

2.1 Third-order FMP Filter

The third-order FMP filter is a constant gain, three-state tracking filter. The three-state vector includes position, velocity and acceleration. The acceleration is assumed to be constant and includes zero mean white Gaussian noise.

Mahafza et al.(2004) put forward that the algorithm involved two major stages of computations, that is, prediction and smoothing. Equations (2.1) - (2.3) are the prediction Equations for position, velocity and acceleration respectively where they are updated from the estimated state thereby lowering the estimation error. Equations (2.4) - (2.6) are the smoothing equations which are computed by adding a weighted difference between the observed and the predicted position to the forecast state.

Prediction step is shown in Equations (2.1) - (2.3):

$$p_{p(n+1)} = p_{s(n)} + T v_{s(n)} + \frac{T^2}{2} a_{s(n)}.$$  \hspace{1cm} (2.1)

$$v_{p(n+1)} = v_{s(n)} + T a_{s(n)}.$$ \hspace{1cm} (2.2)

$$a_{p(n+1)} = a_{s(n)}.$$ \hspace{1cm} (2.3)
Smoothing step:

\[ p_s(n) = p_{p(n)} + \alpha(p_{o(n)} - p_{p(n)}) \tag{2.4} \]
\[ v_s(n) = v_{s(n-1)} + T a_{s(n-1)} + \frac{\beta}{T}(p_{o(n)} - p_{p(n)}) \tag{2.5} \]
\[ a_s(n) = a_{s(n-1)} + \frac{2\gamma}{T^2}(p_{o(n)} - p_{p(n)}) \tag{2.6} \]

where the subscripts \( o, p \) and \( s \) denote the observed, predicted and smoothed state parameters respectively.

\( p, v \) and \( a \) are the target’s position, velocity and acceleration respectively,

\( T \) is the sampling interval of 3 s which coincides with the radar scan rate of 20 rpm.

\( n \) is the \( n^{th} \) sample step.

The selection of the smoothing coefficient is an important design consideration as it directly affects the stability of the output data, error reduction capability and other key design parameters. The FMP filter model has three real roots and represents the filter minimizing the discounted old data least squares error for a constantly accelerating target whereby the position, velocity and acceleration gain coefficients are determined by the damping parameter, \( \xi \), as shown in Equations (2.7) – (2.9):
\[ \alpha = 1 - \xi^3. \] 
\[ \beta = 1.5(1 - \xi)^2(1 + \xi). \] 
\[ \gamma = (1 - \xi)^3. \] 

where \( \alpha, \beta \) and \( \gamma \) are the position, velocity and acceleration smoothing coefficients, \( \xi \) is the damping parameter whose value lies in the interval \([0, 1]\).

Equations (2.7) - (2.9) show that the gains of FMP filter are determined by only one parameter, \( \xi \). Therefore the performance of filter is influenced by the value of \( \xi \).

2.2 Fourth-order FMP Filter

The fourth-order FMP filter is extended from third-order FMP filter. Based on the two major stages of third-order FMP filter algorithm, the prediction stage and smoothing stage can be derived as Equations (2.10) - (2.17). Equations (2.10) - (2.13) are the prediction equations for position, velocity, acceleration and jerk respectively where they are updated from the estimated state thereby lowering the tracking error Equations (2.14) - (2.17) are the smoothing equations which are computed by adding a weighted difference between the observed and the predicted position to the forecast state.

Prediction step as shown in Equations (2.10) - (2.13):
\begin{align}
p_{p(n+1)} &= p_s(n) + T v_{s(n)} + \frac{T^2}{2} a_{s(n)} + \frac{T^3}{6} j_{s(n)}, \quad (2.10) \\
v_{p(n+1)} &= v_s(n) + T a_{s(n)} + \frac{T^2}{2} j_{s(n)}, \quad (2.11) \\
a_{p(n+1)} &= a_s(n) + T j_{s(n)}, \quad (2.12) \\
j_{p(n+1)} &= j_{s(n)}. \quad (2.13)
\end{align}

Smoothing step:

\begin{align}
p_s(n) &= p_p(n) + \alpha (p_o(n) - p_p(n)), \quad (2.14) \\
v_s(n) &= v_p(n) + \frac{\beta}{T} (p_o(n) - p_p(n)), \quad (2.15) \\
a_s(n) &= a_p(n) + \frac{2\gamma}{T^2} (p_o(n) - p_p(n)), \quad (2.16) \\
j_s(n) &= j_p(n) + \frac{6\eta}{T^3} (p_o(n) - p_p(n)). \quad (2.17)
\end{align}

where the subscripts \( o, p \) and \( s \) denote the observed, predicted and smoothed state parameters respectively,

\( p, v, a \) and \( j \) are the target’s position, velocity, acceleration and jerk respectively,

\( T \) is the sampling interval of \( 3 \) s,

\( n \) is the \( n^{th} \) sample step.

The filter weight constants, \( \alpha, \beta, \gamma \) and \( \eta \), are computed by the FMP filter algorithm as shown in Equations (2.18) – (2.21). The \( \xi \) represents the discounting factor that minimizes the least squares error for a constant jerk input model of target dynamics.
\[ \alpha = 1 - \xi^4. \]  
\[ \beta = \frac{1}{6} (1 - \xi)^2 (11 + 14\xi + 11\xi^2). \]  
\[ \gamma = 2(1 - \xi)^2 (1 + \xi), \]  
\[ \eta = \frac{1}{6} (1 - \xi)^4. \]

where \( \alpha, \beta \) and \( \gamma \) are the position, velocity and acceleration smoothing coefficients, \( \xi \) is the damping parameter whose value lies in the interval \([0, 1]\).

Compared with third-order filter algorithm, the fourth-order filter algorithm introduces the fourth state jerk to correct the sudden change of speed to track high mobility warships which have quite maneuverable.

2.3 Superiority of FMP Filter

As it exists numerous of \( \alpha - \beta - \gamma \) tracking filter algorithms and Kalman filter, this section introduces the commonly used \( \alpha - \beta - \gamma \) filter algorithms and compares the performance of them.

2.3.1 Comparison of FMP Filter and Other Filter Algorithms

2.3.1.1 Benedict–Bordner Model

The optimal filter is obtained when;

\[ \beta = \frac{\alpha^2}{2-\alpha}. \]  

The design of this filter does not specify the optimal position smoothing
coefficient, $\alpha$, hence it is chosen based on the system application. It is proposed to vary $\alpha$ with observed high frequency power fluctuations of the tracking error residual or the innovation, $(p_{\alpha(n)} - p_{\beta(n)})$.

The Benedict-Bordner filter coefficient relationship becomes an optimal third order tracking filter when the following condition is satisfied as shown by Simpson (1963):

$$2\beta - \alpha(\alpha + \beta + \frac{\gamma}{2}) = 0. \quad (2.23)$$

In this study, the position smoothing coefficient, $\alpha$, was determined by plotting it against the tracking error as shown below in Figure (2.1). The interval evaluated was selected based on the stability constraints given by Jury (1964) for the $\alpha - \beta - \gamma$ tracking filter. The value of the $\alpha$ that best reduced the innovation was found to be 0.86.

![Figure 2.1 Residual error between observed and predicted positions against corresponding value of $\alpha$](image)

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2.3.1.2 Filter Gain Coefficients Selection Using the Gray & Murray Filter

This filter is an extension of the Kalata filter coefficients relationship which employs the tracking index to compute a damping parameter which is consequently used to calculate the position smoothing coefficient, $\alpha$. The tracking index is given by the following relationship (2.24):

$$A = \frac{T^2 \sigma_w}{\sigma_v},$$  \hspace{1cm} (2.24)

where $T$ is the sampling interval and $\sigma_w$ and $\sigma_v$ are standard deviation of the maneuverability and measurement noise respectively.

The damping parameter, $r$, is computed as shown in Equation (2.25). The position, velocity and acceleration gain parameters, $\alpha$, $\beta$, and $\gamma$, are obtained explicitly as shown in Equations (2.26) - (2.28):

$$r = \frac{(4 + A) - \sqrt{8A + A^2}}{4},$$  \hspace{1cm} (2.25)

$$\alpha = 1 - r^2,$$  \hspace{1cm} (2.26)

$$\beta = 2(2 - \alpha) - 4 \sqrt{1 - \alpha}.$$  \hspace{1cm} (2.27)

$$\gamma = \frac{\beta^2}{2\alpha}.$$  \hspace{1cm} (2.28)

The maneuverability and measurement noise variances were determined by changing the $\sigma^2_w$ and $\sigma^2_v$ error variances while simultaneously feeding the measurement data to the filter for each error variance. The output was then used to compute cumulative positional error which was then
plotted against corresponding error variances. The purpose of this procedure was to identify the error variance coefficient corresponding to the least error. From Figures (2.2) – (2.5), the values of the process and measurement error variance coefficients corresponding to the minimum residual error are 1 and $10^{-3}$ respectively. Consequently the respective standard deviations are $\sigma_w=0.03162$ and $\sigma_1=1$.

The tracking index was, therefore, computed as $\Lambda=0.2846$ and $r=0.6873$. The smoothing coefficients are then obtained from Equations (2.26) – (2.28).

![Figure 2.2](image.png)

*Figure 2.2* Cumulative error difference between observed and predicted positions against maneuverability error variance
Figure 2.3 Cumulative error difference between true and smoothed positions against maneuverability error variance.

Figure 2.4 Cumulative error difference between observed and predicted positions against measurement error variance.
2.3.1.3 Simulation

(1) Target’s dynamics input model

The simulation tests were carried out on a target ship moving at the initial speed of 50 m/s. A sample of $n=1000$ data points was investigated at sampling interval time of 3 s which coincides with the radar scan rate of 20 rpm. The target’s initial position was (573, 1038.4) on the Cartesian coordinates as observed from the radar range measurements.

The input model employed to generate the target dynamics is as shown in Equations (2.29) & (2.30):
\[ x_t = a[10\sin (1.2t) + 7\cos (0.99t) + 8\sin (0.7t) + 6\cos (2t) + 9\sin (3t) + 5\cos (3t)] + 10t, \] (2.29)

\[ y_t = b[20\cos (0.3t) + 22\sin (2t)]. \] (2.30)

where \( t \) is the time of \( n^{th} \) step, \( t = nT \), \( T \) is the sampling interval, \( n \) is the \( n^{th} \) sample step. Parameters \( a \) and \( b \) are used to control the speed of input motion. In order to make sure the speed of motion is in the interval of \([0, 50]\), \( a \) set as 30 and \( b \) is set as 50.

The resulting data was then sampled at intervals of 3 seconds to obtain the true trajectory of the target as shown below in Figure (2.6). The curve has a sudden change of course, it follows the dynamic property of warship.

\[ \text{North-South position, m} \]
\[ \text{East-West position, m} \]

\textbf{Figure 2.6} Input model of comparison of filters

(2) Noise Addition

The observation measurement obtained from the radar sensor contains
an error which was accounted for by corrupting the true positions with zero mean random white Gaussian noise of which standard deviation, $\sigma$, is 10 m. Figures (2.7) & (2.8) show the error distribution in the observation.

**Figure 2.7** East-West error in the observation of comparison of filters
Figure 2.8 North–south error in the observation of comparison of filters

(3) $\alpha - \beta - \gamma$ filter smoothing coefficients

The weights of filters were computed using different methods as discussed in Section 2. Table 2.1 shows a summary of the smoothing constants as obtained from the various methods.

Table 2.1 Filter weights computed from different methods

<table>
<thead>
<tr>
<th>Filter type</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gray and Murray</td>
<td>0.5277</td>
<td>0.1956</td>
<td>0.0101</td>
</tr>
<tr>
<td>Benedict–Bordner</td>
<td>0.8600</td>
<td>0.6488</td>
<td>$4.4409 \times 10^{-16}$</td>
</tr>
<tr>
<td>FMP filter</td>
<td>0.7379</td>
<td>0.3188</td>
<td>0.0467</td>
</tr>
</tbody>
</table>

2.3.1.4. Performance Comparison of the Filters

The three filters under investigation in this study were compared based on the ability to follow a highly maneuvering target marred by randomly
changing accelerations along its trajectory. The capability of total transient error reduction was also examined for the three filters. The total error in this case is obtained from summations of the differences between the predicted position and true position, and the differences between the estimated position and true position.

Figures (2.9) - (2.11) show target trajectories obtained using the FMP filter, the Benedict Bordner and the Gray and Murray filter respectively. The Gray & Murray filter positional trajectories shown in Figure (2.11) appear to follow the target quite steadily compared to the FMP filter shown in Figure (2.9) which can be seen to have a bit of overshooting at various points along the target’s curves. As for the Benedict-Bordner filter, shown in Figure (2.10), the filter performs worse than the other two filters based on the visibly clear jerky motion, an indication of its inability to follow this kind of target maneuver efficiently.

![Graph of target's position](image)

**Figure 2.9** Target’s true, observed, predicted and smoothed position ($\xi=0.64$), FMP filter
Figure 2.10 Target’s true, observed, predicted and smoothed position, Benedict-Bordner filter

Figure 2.11 Target’s true, observed, predicted and smoothed position, Gray & Murray filter
Figures (2.12) - (2.14) show the prediction errors resulting from the FMP filter, the Benedict-Bordner filter and the Gray & Murray filter respectively. Figures (2.15) - (2.17) are the smoothing errors obtained from the FMP filter, the Benedict-Bordner filter and the Gray & Murray filter respectively.

**Figure 2.12** Prediction tracking error of FMP filter in comparison of filters.
Figure 2.13 Prediction tracking error of Benedict-Bordner filter in comparison of filters

Figure 2.14 Prediction tracking error of Gray and Murray filter in comparison of filters
Figure 2.15 Smoothing tracking error of FMP filter in comparison of filters

Figure 2.16 Smoothing tracking error of Benedict-Bordner filter in comparison of filters
Figure 2.17 Smoothing tracking error of Gray and Murray filter in comparison of filters

These results show that the FMP filter has a higher accuracy in both prediction and smoothing of the position of the target warship, followed by the Gray & Murray filter. The Benedict-Bordner filter performs the worst in terms of noise reduction for both prediction and smoothing. These performance results are summarized below in Table 2.2.

Table 2.2 Comparison of prediction and smoothing accuracy of different filters

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error, $m$</th>
<th>Smoothing tracking error, $m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMP filter</td>
<td>19,622</td>
<td>10,653</td>
</tr>
<tr>
<td>Benedict-Bordner</td>
<td>26,326</td>
<td>11,677</td>
</tr>
<tr>
<td>Gray &amp; Murray</td>
<td>21,071</td>
<td>11,693</td>
</tr>
</tbody>
</table>
Of the three filters, the Benedict-Bordner filter performed the worst as the resulting trajectories were characterized by overshooting at various points of the target’s curves.

The FMP filter, on the other hand, performed efficiently in terms of noise reduction in both prediction and smoothing which is visibly clear from the high accuracy obtained compared to the Gray and Murray filter. In addition to demonstrating a good capability of following the maneuvering target with ease and steadiness, the FMP was also easy to implement due to its simplicity and low computational load. However, the Gray & Murray filter depicted a better transient response which was visible from the obtained smooth curves of the position trajectories indicating a higher efficiency in following the highly maneuvering target.

2.3.2 Comparison of FMP Filter and Kalman Filter

2.3.2.1 Kalman Filter Algorithm

The Kalman filter is a recursive filter that requires very little data storage as only the incoming data information is used and therefore does not store up previous information. The Kalman gain is also computed recursively.

The Kalman filter model assumes that the state of a system at \(n\) evolved from the prior state at step \(n - 1\) as shown in Equation (2.31):

\[
X_n = FX_{n-1} + w_n, \tag{2.31}
\]

where

\(n\) is the \(n^{th}\) sample step.
The state vector $X$ containing the position, velocity and acceleration parameters as shown follows:

$$
X_n = \begin{bmatrix}
    x_n \\
    y_n \\
    \vdots \\
    x_n \\
    y_n \\
    \vdots \\
    x_n \\
    y_n
\end{bmatrix}
$$

(2.32)

and the state transition matrix is shown as $F$:

$$
F = \begin{bmatrix}
    1 & 0 & T & 0 & T^2/2 & 0 \\
    0 & 1 & 0 & T & 0 & T^2/2 \\
    0 & 0 & 1 & 0 & T & 0 \\
    0 & 0 & 0 & 1 & 0 & T \\
    0 & 0 & 0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

(2.33)

$w_n$ is the process noise with zero mean and standard deviation $\sigma_w$ and $T$ is the sampling interval.
The target measurement equation is given by

$$Z_n = HX_n + v_n,$$  \hspace{1cm} \text{(2.34)}

where $Z_n$ is the measurement vector which comprises only the position since in this study observation is made on the position only,

$v_n$ is the measurement error with zero mean and standard deviation $\sigma_v$,

$H$ is the transformation matrix that maps the true state space into the observed space given by:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$$  \hspace{1cm} \text{(2.35)}

The process of Kalman filter tracking algorithm involves 2 steps:

\textbf{1. Prediction step:}

Equation (2.36) is the predicted state covariance matrix of the process noise $w_n$ and it depicts the accuracy of predicting the target's state at $n+1$ based on the state values obtained at step $n$.

$$P_{n+1} = FP_nF^T + Q_n.$$  \hspace{1cm} \text{(2.36)}

$Q_n$ is the covariance matrix of the dynamic model driving noise vector, $w_n$.

$T$ is the sampling interval,

$n$ is the $n^{th}$ sample step.

$P_n$ is the state covariance matrix at $t$. 
II. Correction step:

Equation (2.37) is the Kalman filtering equation as it computes the updated estimate of the current state of the target. Equation (2.38) is the updated estimate of the state covariance matrix.

\[
\begin{align*}
\hat{X}_n &= X_n + K_n y_n, \\
\hat{P}_n &= (I - K_n H) P_n.
\end{align*}
\]

(2.37) \hspace{1cm} (2.38)

where \( K_n = P_n H \Sigma_n^{-1} \); Kalman gain at step \( n \),

\( y_n = Z_n - HX_n \); Residual at step \( n \),

\( S_n = HP_n H^T + R_n \); Residual covariance,

\( ^\wedge \) denotes the estimated state.

The design of the Kalman filter enables it to easily adapt to changes in noise or sampling intervals while still maintaining optimality since it computes the gains iteratively. This is, however, not the case with the \( \alpha-\beta-\gamma \) filter whose gains remain fixed at known values. This in turn reduces the computational complexities hence giving the \( \alpha-\beta-\gamma \) filter an advantage over the Kalman filter.

2.3.2.2 Simulation of Kalman Filter and FMP Filter

The input model of FMP is the same model as illustrated in section 2.3.1.3.

(1) Noise Addition

The observed position is the output obtained from the radar
measurements and therefore includes an error. In this study, the noisy observation was obtained by corrupting the true state with zero mean random white Gaussian noise of which standard deviation, \( \sigma \), is 10 m. Figure (2.18) and Figure (2.19) show the error distribution in the observation.

![Figure 2.18 East-West error in the observation of comparison of Kalman filter and FMP filter](image)

**Figure 2.18** East–West error in the observation of comparison of Kalman filter and FMP filter
Figure 2.19 North-South error in the observation of comparison of Kalman filter and FMP filter

(2) The $\alpha$, $\beta$ and $\gamma$ Selection

The filtering coefficients, $\alpha$, $\beta$ and $\gamma$, were chosen based on the optimal value of the damping parameter $\xi$. As mentioned previously, it suggested that the damping parameter $\xi$ should be set as 0.64 for a maneuvering target with an initial speed of 50.4 m/s. Equations (2.7) - (2.9) were then employed to compute the optimal filtering coefficients. Therefore, the $\alpha-\beta-\gamma$ filter used in this study for comparison is an optimal filter.

(3) Kalman filter Tuning

The $R$ matrix shows the accuracy of the radar measurement. Hence, it is the covariance matrix of the measurement error, $v$, with a variance of $\sigma_v^2$. The matrix $Q$, on the other hand, reflects the uncertainty in the target’s trajectory and therefore it is the process noise, $w$, covariance matrix with a variance of $\sigma_w^2$. Since the effects of both $R$ and $Q$ are
negatively correlated, the matrices need to be carefully selected and tuned to avoid divergence of the filter estimates rendering them useless. In addition, given that the $R$ and $Q$ matrix in this study are a fixed value throughout the filtering process, the initial choice of both covariance matrices is crucial in ensuring a good performance of the filter.

The tuning process in this study was achieved by changing the $Q$ and $R$ covariance matrices while simultaneously feeding the measurement data to the filter for each covariance matrix coefficient. The output was then used to compute positional error which was then plotted against corresponding covariance matrix coefficients. The purpose of this was to identify the covariance matrix coefficient corresponding to the least error. From the Figures (2.20) - (2.23) the values of $R$ and $Q$ covariance matrix coefficients corresponding to the minimum residual error are 1 and $10^{-3}$ respectively.

![Graph showing cumulative error difference between observed and predicted positions against $R$ matrix coefficient](image)

**Figure 2.20** Cumulative error difference between observed and predicted positions against $R$ matrix coefficient
Figure 2.21 Cumulative error difference between true and smoothed positions against $R$ matrix coefficient.

Figure 2.22 Cumulative error difference between observed and predicted positions against $Q$ matrix coefficient.
2.3.2.3 Result Comparison of FMP Filter and Kalman Filter

Comparison of the two filters' performances was made based on the filter's capability to minimize the noise level and ability to follow the randomly maneuvering target warship. This was done by comparing the size of the error in the smoothed and predicted positions which are smoothing and prediction error respectively. Smoothing error is obtained by computing the deviation of the estimation from the true position for each sample. Similarly, prediction error indicates how far the predicted position deviates from the true position.

Figures (2.24) - (2.27) show the enlarged views of true, observed, predicted and smoothed position trajectories of FMP filter and Kalman filter. The enlarged part of Figures (2.24) & (2.25) are relatively high manoeuvring parts which lie in the interval of [2700, 3700] in X-axis. From
the curves, the FMP filter seems to easily follow the maneuvering target warship with greater sensitivity as indicated by the steadiness in the predicted and smoothed trajectories. On the contrary, the predicted and smoothed curves resulting from the Kalman filter are marred with erratic changes and overshooting at various points on the trajectories. Figures (2.26) & (2.27) show the enlarged views of tracking result of two filters which lie in the interval of [15,400, 16,200] in X-axis. The motion of this part is relatively smooth without high manoeuvring. As shown in these two figures, the overshooting of FMP filter is a little higher than that of Kalman filter, which indicates that the stability of Kalman filter is better when tracking smoothed motion.

![Figure 2.24 Enlarged view of FMP filter result trajectories lies in the interval [2700, 3700] in the X-axis](image-url)
Figure 2.25 Enlarged view of Kalman filter result trajectories lies in the interval [2700, 3700] in the X-axis.

Figure 2.26 Enlarged view of FMP filter result trajectories lies in the interval [15400, 16200] in the X-axis.
Figure 2.27 Enlarged view of Kalman filter result trajectories lies in the interval [15400, 16200] in the X-axis.

Figure (2.28) & Figure (2.29) show the prediction errors resulting from the FMP filter and the Kalman filter respectively. Figure (2.30) & Figure (2.31) are the smoothing errors obtained from the FMP filter and the Kalman filter respectively. These results show that the Kalman filter has a slightly higher accuracy in both prediction and smoothing of the position of the target warship than the FMP filter.
Figure 2.28 Prediction tracking error of FMP filter of comparison of Kalman filter and FMP filter.

Figure 2.29 Prediction tracking error of Kalman filter of comparison of Kalman filter and FMP filter.
Figure 2.30 Smoothing tracking error of FMP filter of comparison of Kalman filter and FMP filter.

Figure 2.31 Smoothing tracking error of Kalman filter of comparison of Kalman filter and FMP filter.
2.3.2.4 Analysis of Simulation Result

In this section, two filters have been considered for comparison in their performance. The results obtained depict that both filters, when properly designed and tuned, are capable of tracking the high mobility warship with some degree of accuracy. As shown in Table 2.3, the comparison result indicates that the Kalman filter has a little higher accuracy than FMP filter in both prediction and smoothing of the target’s position. On the other hand, the standard deviation of prediction tracking error and smoothing tracking error of Kalman filter is higher than FMP filter, which means that the stability of FMP filter is better than Kalman filter. The Figure (2.25) – (2.28) show the tracking result of two different results. The Kalman filter performs good when tracking relatively smoothed motion, but the result shows Kalman filter has a higher rate of overshooting than FMP filter when tracking high maneuvering motion. The FMP filter with smoothed and predicted trajectories are steady throughout the whole sample indicating its ability to respond well to the target’s maneuvers.

Table 2.3 Comparison of the tracking performance of FMP filter and Kalman filter

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Prediction tracking error (m)</th>
<th>Smoothing tracking Error (m)</th>
<th>Std. of Prediction tracking error</th>
<th>Std. of Smoothing tracking Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>FMP filter</td>
<td>19,622.41</td>
<td>10,657.31</td>
<td>9.45</td>
<td>5.32</td>
</tr>
<tr>
<td>Kalman filter</td>
<td>19,104.04</td>
<td>10,491.77</td>
<td>9.77</td>
<td>5.45</td>
</tr>
</tbody>
</table>

Table 2.4 shows the run times of Kalman filter and FMP filter. The mean value of run times of Kalman filter of 10 times simulation is 1.833 ms,
30.5 times slower than FMP filter. Meanwhile, the standard deviation of run times of Kalman filter is larger than FMP filter, which means that the stability of time consuming of Kalman filter is relatively lower.

**Table 2.4** Comparison of run times of Kalman filter and FMP filter (unit: \( ms \))

<table>
<thead>
<tr>
<th>Simulation Number</th>
<th>Kalman filter</th>
<th>FMP filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.33333333</td>
<td>0.00046178</td>
</tr>
<tr>
<td>2</td>
<td>0.66666667</td>
<td>0.00046478</td>
</tr>
<tr>
<td>3</td>
<td>1.00000000</td>
<td>0.00045911</td>
</tr>
<tr>
<td>4</td>
<td>1.33333333</td>
<td>0.00046344</td>
</tr>
<tr>
<td>5</td>
<td>1.66666667</td>
<td>0.00046033</td>
</tr>
<tr>
<td>6</td>
<td>2.00000000</td>
<td>0.00046133</td>
</tr>
<tr>
<td>7</td>
<td>2.33333333</td>
<td>0.00046456</td>
</tr>
<tr>
<td>8</td>
<td>2.66666667</td>
<td>0.00045622</td>
</tr>
<tr>
<td>9</td>
<td>3.00000000</td>
<td>0.00045967</td>
</tr>
<tr>
<td>10</td>
<td>3.33333333</td>
<td>0.00046011</td>
</tr>
<tr>
<td>Mean</td>
<td>1.83333333</td>
<td>0.00046113</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.95742711</td>
<td>0.00000251</td>
</tr>
</tbody>
</table>

Compared with Kalman filter, the FMP filter performs higher accuracy of tracking high mobility motion and has less computations even though the overall tracking accuracy of it is a little lower. As high mobility warship has high maneuvering motion especially under emergency situation, FMP
filter is recommended to adopt for tracking.

2.4 Summary of Chapter 2

This Chapter introduced the mathematical model of third-order FMP filter and fourth-order FMP filter, and compared the tracking performance of FMP filter with the performance of other filter algorithms and Kalman filter algorithm. The superiority of FMP filter can be shown as follows:

1. Compared with commonly used benedict-bordner filter algorithm and Gray&Murray filter algorithm, the FMP filter has the highest tracking performance in prediction and smoothing.

2. FMP filter has another advantage that it is easy to implement due to its simplicity and low computational load.

3. Even though the Kalman filter has a little higher tracking accuracy, the simulation results show that the Kalman filter appears to have a high rate of overshooting when tracking high mobility motion, hence it is relatively unstable compared with FMP filter.

4. FMP filter is also easier to implementation and performs more competitively than Kanlam filter.

As previously mentioned, the FMP filter has higher tracking accuracy and also has lower computational load. This study, therefore, uses FMP filter to explore the tracking performance in different cases of tracking high mobility warships.
Chapter 3 Third-order FMP Filter to Track High Mobility Warship

In order to simulate the high mobility motion, the study uses the combined trajectory motion which is generated by sine-cosine equations. This chapter optimizes the third-order FMP filter to track high mobility target, using the cases of own ship are static and motional. Meanwhile, the speed-dependent third-order FMP filter algorithms are also illustrated in this chapter.

3.1 Third-order FMP Filter to Track High Mobility Target

Warship has excellent performance in sudden changes of speed and course, it is difficult to track. This Section will discuss the performance of FMP filter to track realistic motion which is generated by combined trajectory motion. The input model of FMP filter is the same model as illustrated in section 2.3.1.

3.1.1 Noise Addition

The observed position is the output obtained from the radar measurements and therefore includes an error. In this study, the noisy observation was obtained by corrupting the true state with zero mean random white Gaussian noise of which standard deviation, $\sigma$, is 10 m. Figure (3.1) and Figure (3.2) show the error distribution in the observation.
Figure 3.1 East–West error in the observation of third-order FMP filter to track high mobility target.

Figure 3.2 North–South error in the observation of third-order FMP filter to track high mobility target.
3.1.2 Determination of Optimal $\xi$

Figure (3.3) shows the enlarged view of filter result of true, observed, predicted and smoothed position trajectories lies in the interval [2700, 3700] in the X-axis. Figure (3.4) shows the prediction tracking error which stands for the difference between true position and predicted position while Figure (3.5) shows the smoothing tracking error which stands for the difference between true and smoothed position. Afterwards, set $\xi$ varying for 0 to 1 with step of 0.01 and calculated the total prediction tracking error and total smoothing tracking error of each step.

![Figure 3.3](Image)

**Figure 3.3** Enlarged view of tracking result($\xi=0.64$) of third-order FMP filter to track high mobility target
Figure 3.4 Prediction tracking error of third-order FMP filter to track high mobility target

Figure 3.5 Smoothing tracking error of third-order FMP filter to track high mobility target
Figure 3.6 Sum of total prediction tracking error and total smoothing tracking error while \( \xi \) varying from 0 to 1

Figure (3.6) shows the sum of total prediction tracking error and total smoothing tracking error when \( \xi \) value varying from 0 to 1. The curves rapidly decrease and reach the minimum, and then they rapidly increase. The corresponding \( \xi \) value of minimum of curve is the optimal value for the filter. In this case the damping parameter, \( \xi \) was selected as 0.64 for prediction purpose.

3.2 Third-order FMP filter to Track High Mobility Motion under the Condition of both Own Ship and Target in Motion

In previous Section, third-order FMP filter is optimized to track combined trigonometric function motion on condition that own ship is motionless. This Section will discuss much more realistic case that the own ship is also in motion.
3.2.1 Input Model of Own Ship and Target

(1) Input model of target

\[ x_n = a[10\sin(1.2n) + 7\cos(0.99n) + 8\sin(0.7n) + 6\cos(2n) + 9\sin(3n) + 5\cos(3n)] + 10n, \]  
\[ y_n = b[20\cos(0.3n) + 22\sin(2n)] + 3n. \]  
where \( a \) and \( b \) are constants that serve to control the velocity of target in the input motion model. In this case, \( a \) is set to 30 while \( b \) is set to 50.

(2) Input model of own ship

\[ x_n = d[10\cos(0.3n) + 11\sin(2n) + 10\cos(0.5n) + 12\sin(2n)] + 2n, \]
\[ y_n = c[2\sin(0.5n) + 20\cos(3n) + 4.3\sin(3n) + 20\cos(1.8n) + 1.5\sin(1.2n) + 20\cos(0.5n)] + 10n. \]  
where \( c \) and \( d \) are constants that serve to control the velocity of own ship in the input motion model. In this case, \( c \) is set to 30 while \( d \) is set to 50.

The target was modeled under the initial conditions laid out in Table 3.1
The resulting data was then sampled at intervals of 3 seconds to give the true trajectory of own ship, target and relative motion as shown below in Figure (3.7).

![Figure 3.7 Input model of target, own ship and relative motion of third order FMP filter to track high mobility target under the condition of both own ship and target in motion](image)

### 3.2.2 Noise Addition

The observed position is the output obtained from the radar measurements and therefore includes an error. In this study, the noisy
observation was obtained by corrupting the true state with zero mean random white Gaussian noise which standard deviation, $\sigma$, is 10 m. Figure (3.8) and Figure (3.9) show the error distribution in the observation.

**Figure 3.8** East-West error in the observation of third order FMP filter to track high mobility target under the condition of both own ship and target in motion.
3.2.3 Determination of Optimal $\zeta$

Figure (3.10) and Figure (3.11) show the enlarged view of true, observed, predicted and smoothed position trajectories lie in the interval [26000, 28000] and [17900, 18500] in the X-axis when $\zeta=0.54$. Figure (3.12) shows the prediction tracking error which stands for the difference between true position and predicted position, while Figure (3.13) shows the smoothing tracking error which stands for the difference between true and smoothed position. Afterwards, set $\zeta$ varying for 0 to 1 with step of 0.01 and calculated the total prediction tracking error and total smoothing tracking error of each step.
Figure 3.10 Enlarged view of tracking result ($\xi = 0.54$) of third order FMP filter to track high mobility target under the condition of both own ship and target in motion.

Figure 3.11 Enlarged view of tracking result ($\xi = 0.54$) of third order FMP filter to track high mobility target under the condition of both own ship and target in motion.
Figure 3.12 Prediction tracking error of third-order FMP filter to track high mobility target under the condition of both in motion.

Figure 3.13 Smoothing tracking error of third-order FMP filter to track high mobility target under the condition of both in motion.
Figure 3.14 Sum of total prediction error and total smoothing error of third-order FMP filter to track high mobility target under the condition of both in motion

Figure (3.14) shows the total prediction tracking error and total smoothing tracking error when $\xi$ value varying from 0 to 1. In this case the damping parameter, $\xi$ was selected as 0.56 for prediction purpose.

3.3 Speed-dependent Third-order FMP Filter to Track High Mobility Target

As mentioned above, third-order FMP was employed to track a high mobility target and it was proven that the filter was capable of following a highly maneuvering target under a high initial speed with a good degree of accuracy. However, it can only obtain the optimal parameter till the target finishes high mobility motion. Pan et al(2016) illustrated that the optimal parameter $\xi$ had an approximate inverse proportionality with the
speed of the target. In the real case, it is urgently needed to find a way which can get auto-matching parameter based on the speed in order to obtain the accurate position as quickly as possible. Therefore, as the optimal parameter has relationship with speed, a speed-dependent optimized filter should be considered to employ to track high mobility warship.

3.3.1 Input Model of Target

The input model of FMP filter is the same model as illustrated in section 2.3.1. As parameters $a$ and $b$ are used to control target’s speed in Equations (2.29 – 2.30), in order to find the performance of filter influenced by speed, $a, b$ are looped from 1 to 150 with step of 5.

3.3.2 Noise Addition

Since the measurements from the radar sensor contain errors, this was taken into account by adding a zero mean random white Gaussian noise with a standard deviation, $\sigma$ of $10 \text{ m}$ to the true position sample. This error distribution in the observation is as shown in Figure (3.15) and Figure (3.16).
Figure 3.15 East-West error in the observation of speed-dependent third order FMP filter to track high mobility target.

Figure 3.16 North-South error in the observation of speed-dependent third order FMP filter to track high mobility target.
3.3.3 Determination of optimal ksi, $\xi$

The filter’s weight constants $\alpha, \beta, \gamma$ values depend on the value of the discounting factor $\xi$. Therefore, in order to obtain the optimal smoothing coefficients, optimization of the FMP filter focuses on adjusting the $\xi$ value experimentally through trial and error method.

The process of optimization begins by computation of the total transient error which is the sum of the squares of the difference between the true trajectory and the predicted trajectory. The purpose of the optimization is to find the discounting factor that minimizes this error, and then to use this information to compute the optimal smoothing constants. This is, therefore, achieved by plotting a range of the discounting factor, which lies in the interval [0, 1] and with a step increase of 0.01, against the corresponding transient error. Thirty simulation tests are then carried out for each $\xi$ and the average obtained. The resulting figure is as shown in Figure (3.17) below. According to this graph, $\xi$ was selected as 0.53 for prediction purpose.
Figure 3.17 Total prediction error and total smoothing error of third-order FMP filter to track high mobility target under the condition of both in motion

3.3.4 Relationship between Speed and Optimal $\xi$

Curve fitting is a method of approximating discrete data with analytical expressions by using continuous curves to approximate the coordinate relationships represented by sets of discrete points. Least squares method is the most commonly used curve fitting method as it is more accurate and practical.

Setting coefficients $a, b$ in Equations (2.29) - (2.30) looped from 1 to 150 with step of 5, after plotting 900 sets of $\xi - v$ points, it is clear to know that the connection exists between $\xi$ and $v$. Since the data between $\xi$ and $v$ in Figure (3.18) are linearly related, it is reasonable to assume an approximation of a linear relation as
\[ \xi = c_1 v^4 + c_2 v^3 + c_3 v^2 + c_4 v + c_5 \]  
(3.5)

Rewriting Equation (4.15) yields

\[
\begin{bmatrix}
  c_1 \\
  c_2 \\
  c_3 \\
  c_4 \\
  c_5
\end{bmatrix} =
\begin{bmatrix}
  v^4 & v^3 & v^2 & v & 1
\end{bmatrix}
\]  
(3.6)

Applying the least squares method to given data points of 900, the estimate yields

\[
\hat{x} = (A^T A)^{-1} A^T \xi
\]  
(3.7a)

where

\[
A = \begin{bmatrix}
  v_1^2 & v_1 & 1 \\
  v_2^2 & v_2 & 1 \\
  \vdots & \vdots & \vdots \\
  v_{900}^2 & v_{900} & 1
\end{bmatrix} \in \mathbb{R}^{900 \times 5}, \quad \xi = \begin{bmatrix}
  \xi_1 \\
  \xi_2 \\
  \vdots \\
  \xi_{900}
\end{bmatrix} \in \mathbb{R}^{900}, \quad \hat{x} = \begin{bmatrix}
  \hat{c}_1 \\
  \hat{c}_2 \\
  \hat{c}_3 \\
  \hat{c}_4 \\
  \hat{c}_5
\end{bmatrix} \in \mathbb{R}^{5}
\]  
(3.7b)

Using the obtained 900 sets of average speed and optimal \( \xi \), the curve fitting result can be obtained as Figure (4.29).
Figure 3.18 Curve fitting result of speed-dependent third-order FMP filter to track high mobility motion

With the result of 900 sets of data, the Equation (3.5) can be obtained as follows:

\[
\xi = 2.8 \times 10^{-9} \cdot v^4 - 1.535 \times 10^{-6} \cdot v^3 + 2.25 \times 10^{-4} \cdot v^2 \\
- 1.39 \times 10^{-2} \cdot v + 0.831
\]  

(3.8)

As shown in the table 3.2, it is visibly clear that the speed-dependent third-order FMP filter has a higher performance than the classic third-order FMP filter. Quantitatively, the total prediction tracking error and total smoothing tracking error are used to evaluate the performance of tracking filter. The prediction tracking error of speed-dependent third-order FMP filter amounts to 18,632.96 m while the classic third-order FMP filter depicts its optimal performance with prediction tracking error
resulting to 19622.41 m. The accuracy in tracking is therefore improved by 989.45 m equivalent to 5% higher. Similarly, the smoothing accuracy is increased by 161.05 m by employing the speed-dependent third-order FMP filter. Meanwhile, the standard deviation of prediction tracking error and standard deviation of smoothing tracking error of speed-dependent FMP filter are obviously smaller than them of classic FMP filter, which indicates that the stability of speed-dependent FMP filter is much more better.

**Table 3.2** Performance comparison of speed-dependent third-order FMP filter and FMP filter to track high mobility motion

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error</th>
<th>Smoothing tracking error</th>
<th>Std. of prediction tracking error</th>
<th>Std. of smoothing tracking error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed-dependent third-order FMP filter</td>
<td>18,632.96</td>
<td>10,496.26</td>
<td>9.45</td>
<td>5.32</td>
</tr>
<tr>
<td>Classic FMP filter</td>
<td>19,622.41</td>
<td>10,657.31</td>
<td>10.16</td>
<td>5.64</td>
</tr>
</tbody>
</table>

3.4 Speed-dependent Third-order FMP Filter to Track High Mobility Target under the Condition of both Own Ship and Target in Motion

3.4.1 Input Model

The input model of FMP filter is the same model as illustrated in section 3.2. As parameters \( a, b, c, d \) are used to control target’s speed in Equations (3.1) – (3.4), in order to find the performance of filter influenced by speed, \( a \) and \( b \) are looped from 1 to 150 with step of 15.
3.4.2 Noise Addition

Since the measurements from the radar sensor contain errors, this was taken into account by adding a zero mean random white Gaussian noise with a standard deviation, $\sigma$ of 10 $m$ to the true position sample. This error distribution in the observation is as shown in Figure (3.19) and Figure (3.20).

![Figure 3.19 East-West error in the observation of speed-dependent third order FMP filter to track high mobility target under the condition of both own ship and target in motion](image)
Figure 3.20 North-South error in the observation of speed-dependent 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.

3.4.3 Determination of Optimal ksi, $\xi$

The filter weight constants $\alpha$, $\beta$, $\gamma$ values depend on the value of the discounting factor $\xi$. Therefore, in order to obtain the optimal smoothing coefficients, optimization of the FMP filter focuses on adjusting the $\xi$ value through trial and error method.

The process of optimization begins by computation of the total transient error which is the sum of the squares of the difference between the true trajectory and the predicted trajectory. The purpose of the optimization is to find the discounting factor that minimizes this error then to use this information to compute the optimal smoothing constants. This is, therefore, achieved by plotting a range of the discounting factor, which lies in the interval $[0, 1]$ and with a step increase of 0.01, against the corresponding transient error. Thirty simulation tests are then carried out for each $\xi$ and
the average obtained. The resulting figure is as shown in Figure (3.21) below. According to this graph, \( \xi \) was selected as 0.57 for prediction purpose.

![Figure 3.21](image)

**Figure 3.21** Total prediction error and total smoothing error of third-order FMP filter to track high mobility target under the condition of both in motion

### 3.4.4 Relationship between Speed and Optimal \( \xi \)

Assume an approximation of a linear relation as

\[
\xi = c_1 v^4 + c_2 v^3 + c_3 v^2 + c_4 v + c_5
\]  

(3.9)
Curve fitting result of speed-dependent third-order FMP filter to track high mobility target under the condition of both own ship and target in motion

With the result of 10000 sets of data, the Equation (4.10) can be obtained as follows:

$$\xi = 2.35 \times 10^{-8} \cdot v^3 + 2.62 \times 10^{-5} \cdot v^2 - 0.0048 \cdot v + 0.6777$$  \hspace{1cm} (3.10)

As shown in Table 3.3, it is visibly clear that the speed-dependent third-order FMP filter has a higher performance than the classic third-order FMP filter. Quantitatively, the total prediction tracking error and total smoothing tracking error are used to evaluate the performance of tracking filter. The prediction tracking error of speed-dependent third-order FMP filter amounts to 21,059.89 m while the classic third-order FMP filter depicts its optimal performance with prediction tracking error resulting to 21,461.64 m. The accuracy in prediction is, therefore,
improved by 401.75 m. Similarly, the smoothing accuracy is increased by 143.59 m by employing the speed-dependent third-order FMP filter. Meanwhile, the standard deviation of prediction tracking error and the standard deviation of smoothing tracking error of speed-dependent FMP filter are obviously smaller than them of classic FMP filter, which indicates that the stability of speed-dependent FMP filter is much better.

**Table 3.3** Performance comparison of speed-dependent third-order FMP filter and FMP filter to track high mobility target under the condition of both own ship and target in motion

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error</th>
<th>Smoothing tracking error</th>
<th>Std. of prediction tracking error</th>
<th>Std. of smoothing tracking error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed-dependent third-order FMP filter</td>
<td>21059.89</td>
<td>10809.96</td>
<td>10.9548</td>
<td>5.5632</td>
</tr>
<tr>
<td>Classic FMP filter</td>
<td>21461.64</td>
<td>10953.55</td>
<td>11.6147</td>
<td>5.7396</td>
</tr>
</tbody>
</table>

**3.5 Summary of Chapter 3**

This Chapter explores the performance of third-order FMP filter for tracking high mobility target under the condition of own ship is motionless and in motion. The result can be summed as follows:

(1) The $\xi$ value is optimized as 0.64 when tracking trigonometric function combined trajectory motion.

(2) The $\xi$ value is optimized as 0.56 when tracking trigonometric function combined trajectory motion under the high mobility motion of both own ship and target.
(3) Speed-dependent third-order FMP filter algorithm to track high mobility target is developed and it performs higher tracking accuracy and high stability than classic FMP filter.

(4) Speed-dependent third-order FMP filter algorithm to track high mobility target under the condition of own ship and target in motion is developed and it performs higher tracking accuracy and stability than the classic FMP filter.

As the relative motion has sudden changes in speed and acceleration, therefore it is necessary to consider the correction of jerky, the change of acceleration, to improve the tracking accuracy. Hence the next Chapter will discuss the fourth-order FMP filter for tracking high Mobility warship.
Chapter 4 Fourth-order FMP Filter to Track High Mobility Warship

Chapter 3 discussed the performance of third-order FMP filter to track various of motion, and it is mentioned that the third-order FMP filter is powerless to track high mobility motion especially the motion has a sudden change in acceleration. Therefore, this Chapter will discuss the fourth-order FMP filter to track high mobility warship.

4.1 Fourth-order FMP filter to Track Trigonometric Function Combined Trajectory Motion only Target in Motion

The four-state vector of fourth-order FMP filter includes position, velocity, acceleration and jerk, a time derivative of acceleration. The jerk is modelled as a constant and includes zero mean white Gaussian noise.

Based on the two major stages of third-order FMP filter algorithm which were illustrated by Mahafza et al. (2004), the prediction stage and smoothing stage of fourth-order filter can be derived as Equations (2.10) - (2.17). Equations (2.10) - (2.13) are the prediction equations for position, velocity, acceleration and jerk respectively where they are updated from the estimated state thereby lowering the tracking error Equations (2.14) - (2.17) are the smoothing equations which are computed by adding a weighted difference between the observed and the predicted position to the forecast state.

4.1.1 Simulation of Fourth-order FMP Filter

In order to evaluate the performance of fourth-order FMP filter and
third-order FMP filter, this Section uses the same input model of third-order FMP filter to do the simulation. The input model is the same model as illustrated in section 2.3.1.3.

(1) Noise addition

Since the measurements from the radar sensor contain errors, this was taken into account by adding a zero mean random white Gaussian noise with a standard deviation, $\sigma$ of 10 m to the true position sample. This error distribution in the observation is as shown in Figure (4.1) and Figure (4.2).

![Figure 4.1 East-West error in the observation of 4th order FMP filter to track high mobility target](image)
Figure 4.2 North-South error in the observation of 4th order FMP filter to track high mobility target.

(2) Determination of optimal ksi, $\xi$

The filter’s weight constants $\alpha$, $\beta$, $\gamma$ and $\eta$ values depend on the value of the discounting factor, $\xi$ as shown in Equations (2.18) - (2.21). Therefore, in order to obtain the optimal smoothing coefficients, optimization of the FMP filter focuses on adjusting the $\xi$ value experimentally through trial and error method.

The process of optimization begins by computation of the total transient error which is the sum of the squares of the difference between the true trajectory and the predicted trajectory. The purpose of the optimization is to find the discounting factor that minimizes this error then to use this information to compute the optimal smoothing constants. This is, therefore, achieved by plotting a range of the discounting factor, which lies in the interval $[0, 1]$ and with a step increase of 0.01, against the corresponding...
transient error. Thirty simulation tests are then carried out for each $\xi$ and the average is obtained. The resulting figure is as shown in Figure (4.3) below. According to this graph, the value of $\xi$ corresponding to the least residual error is 0.74. Figure (4.4) shows the consistency regarding the optimal $\xi$ and therefore upholds the validity of the results as it indicates a similar value of the optimal discounting factor. They represent plots of the total error difference between the true and predicted trajectories and between the true and smoothed trajectories respectively against corresponding $\xi$ values.

Figure 4.3 Sum of the prediction tracking error of 4th order FMP filter to track high mobility target
4.1.2 Result Analysis of Fourth-order FMP Filter

The performance of the optimal fourth-order FMP filter is evaluated based on its ability to follow the maneuvering target with ease and with an increased reduction of overshooting along the prediction trajectory due to jerky movements. In addition, the filter’s performance is further investigated based on its capability of reduction of tracking error. The performance is then compared with that of the optimal third-order FMP filter under the similar conditions of target motion.

Using the obtained optimal discounting factor ($\xi=0.74$) the optimal filter weights were computed, and consequently the predicted and smoothed trajectory were obtained. Figure (4.5) shows the true, observed, predicted and smoothed trajectories of fourth-order FMP filter. The curve enclosed
in the rectangle is enlarged for clearer viewing as shown in Figure (4.6). The positional trajectories obtained from simulation tests which are carried out under similar conditions with the optimal third-order FMP filter are as shown below in Figure (4.7). From the obtained trajectories, the fourth-order FMP filter can be observed to easily follow the highly maneuvering target with greater sensitivity as indicated by the steadiness in the predicted and smoothed trajectories. In contrast, the predicted and smoothed trajectories resulting from the third-order FMP filter are seen to have erratic changes and overshooting at various points on the trajectories indicating the filter’s inability to respond well to random changes in speed and quick maneuvers particularly for this type of trajectory.

![Tracking result of 4th order FMP filter to track high mobility target](image)

**Figure 4.5** Tracking result of 4th order FMP filter to track high mobility target
Figure 4.6 Enlarged view of tracking result of 4th order FMP filter to track high mobility target

Figure 4.7 Enlarged view of tracking result of 3rd order FMP filter to track high mobility target
**Figure 4.8** Prediction tracking error and smoothing tracking error of 3rd order FMP filter to track high mobility target

**Figure 4.9** Prediction tracking error and smoothing tracking error of 4th order FMP filter to track high mobility target
Figure (4.8) and Figure (4.9) plot the prediction tracking error and smoothing tracking error of third-order FMP filter and fourth-order FMP filter. It is clear that the two filters perform good in tracking high mobility motion. However, after comparing the tracking result of two filters, it is visibly clear that the fourth-order FMP filter has a higher performance than the third-order FMP filter. Quantitatively, the total prediction tracking error and total smoothing tracking error are used to evaluate the performance of two filters. As shown in the table below, the third-order FMP filter demonstrates the best performance when $\xi=0.64$ with the prediction tracking error amounting to 19,622.41 m. On the contrary, when $\xi=0.74$ the fourth-order FMP filter depicts its optimal performance with prediction tracking error resulting to 17,858.93 m. The accuracy in prediction is therefore improved by 1,763.48 m equivalent to 8.99% higher. Similarly, the smoothing accuracy is increased by 430.78 m by employing the fourth-order FMP filter. Meanwhile, the standard deviation of prediction tracking error and smoothing tracking error of fourth-order FMP filter are a little smaller than third-order FMP filter. It indicates that the stability of fourth-order FMP filter is higher than that of third-order FMP filter.
Table 4.2 Performance comparison of 3rd order FMP filter and 4th order FMP filter to track high mobility target

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Optimal $\xi$</th>
<th>Prediction tracking error ($m$)</th>
<th>Smoothing tracking Error ($m$)</th>
<th>Std. of Prediction tracking error</th>
<th>Std. of Smoothing tracking Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>3rd order FMP filter</td>
<td>0.64</td>
<td>19,622.41</td>
<td>10,657.31</td>
<td>10.16</td>
<td>5.64</td>
</tr>
<tr>
<td>4th order FMP filter</td>
<td>0.74</td>
<td>17,858.93</td>
<td>10,226.53</td>
<td>9.09</td>
<td>5.24</td>
</tr>
</tbody>
</table>

Target motions defined by high maneuvering and jerky movements such as high mobility vessels require prompt and accurate predictions of the target dynamics. Standard filters such as the third-order FMP filter are incapable of tracking the randomly changing accelerations with good accuracy since they are designed to track constant acceleration motions only. Therefore, a higher order steady state filter is required to take care of the jerky maneuvers.

The results obtained from simulation tests of the high mobility target model indicate a better performance of the jerk model filter in terms of ability to follow the maneuvering target with a greater sensitivity compared to the third-order FMP filter under the same operating conditions of target motion. This attribute is due to the inclusion of the jerk smoothing constant which is responsible for taking care of the sudden changes in acceleration and target maneuvers. In addition, the tracking accuracy increased by 8.9% for the jerk filter model compared to the third-order FMP filter model. Meanwhile, the stability of fourth-order FMP filter is higher than that of third-order FMP filter.
4.2 Fourth-order FMP Filter to Track Trigonometric Function
Combined Trajectory Motion under the High Mobility Motion of
Both Own Ship and Target

Section 4.1 discussed fourth-order FMP filter to track high mobility
motion on the condition that own ship is motionless. This Section will
discuss more realistic motion which both own ship and target are in
motion.

4.2.1 Input Model of Own Ship and Target

(1) Input model of target

\[
x_t = a[10\sin(1.2t) + 7\cos(0.99t) + 8\sin(0.7t) + 6\cos(2t) \\
+ 9\sin(3t) + 5\cos(3t)] + 10t
\]

where \(x_t\) is the time of \(n^{th}\) step, \(t = nT\), \(T\) is the sampling interval, \(n\) is the
\(n^{th}\) sample step. Parameters \(a\) and \(b\) are constants that serve to control
the velocity of the own ship in the input motion model. In this case, \(a\) is
set to 30 while \(b\) is set to 50.

(2) Input model of own ship

\[
x_n = d[10\cos(0.3t) + 11\sin(2t) + 10\cos(0.5t) + 12\sin(2t)] + 2t
\]

where \(t\) is the time of \(n^{th}\) step, \(t = nT\), \(T\) is the sampling interval, \(n\) is the
\(n^{th}\) sample step. Parameters \(c\) and \(d\) are constants that serve to control
the velocity of own ship in the input motion model. In this case, $c$ is set to 30 while $d$ is set to 50.

The relative motion was modeled under the initial conditions laid out in Table 4.3 below.

**Table 4.3** Initial conditions of relative motion of input model of both own ship and target in motion

<table>
<thead>
<tr>
<th>Position $(x, y)$</th>
<th>Initial speed m/s</th>
<th>sampling interval, s</th>
<th>Sample size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(573, 1038.4)$</td>
<td>50.4</td>
<td>3</td>
<td>1,000</td>
</tr>
</tbody>
</table>

The resulting data was then sampled at intervals of 3 seconds to give the true trajectory of own ship, target and relative motion as shown below in Figure (4.10).

**Figure 4.10** Input model of target, own ship and relative motion of 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion
4.2.2 Noise Addition

The observed position is the output obtained from the radar measurements and therefore includes an error. In this study, the noisy observation was obtained by corrupting the true state with zero mean random white Gaussian noise of which standard deviation, $\sigma$, is 10 m. Figure (4.11) and Figure (4.12) show the error distribution in the observation.

![Figure 4.11 East-West error in the observation of 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion](image)
Figure 4.12 North-South error in the observation of 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.

4.2.3 Determination of Optimal $\xi$

Figure (4.13) shows the enlarged view of true, observed, predicted and smoothed position trajectories lie in the interval [17800, 18600] in the X-axis. Figure (4.14) shows the prediction tracking error which stands for the difference between true position and predicted position while Figure (4.15) shows the smoothing tracking error which stands for the difference between true and smoothed position. Afterwards, $\xi$ is set varying for 0 to 1 with step of 0.01, then the total prediction tracking error and the total smoothing tracking error are calculated of each step.
Figure 4.13 Enlarged view of tracking result of 4\textsuperscript{th} order FMP filter to track high mobility target under the condition of both own ship and target in motion.

Figure 4.14 Prediction tracking error of 4\textsuperscript{th} order FMP filter to track high mobility target under the condition of both own ship and target in motion.
Figure 4.15 Smoothing tracking error of 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.
Figure 4.16 Sum of prediction tracking error and smoothing tracking error of 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.

Figure (4.16) shows the sum of prediction tracking error and sum of smoothing tracking error when $\xi$ value varying from 0 to 1. Both of the curves rapidly decrease and reach the minimum, and then they rapidly increase. The corresponding $\xi$ value of minimum of curve is the optimal value for the filter. In this case the damping parameter, $\xi$, was selected as 0.50 for the prediction purpose.

4.3 Speed-dependent Fourth-order FMP Filter to Track High Mobility Targets

As mentioned above, fourth-order FMP was employed to track a high mobility target and it was proven that the filter was capable of following
a highly maneuvering target under a high initial speed with a good degree of accuracy. However, it can only obtain the optimal parameter till the target finishes high mobility motion. Pan et al (2016) illustrated that the optimal parameter $\xi$ had an approximate inverse proportionality with the speed of the target. In the real case, it is urgently needed to find a way which can get auto-matching parameter based on the speed in order to obtain the accurate position as quickly as possible. Therefore, as the optimal parameter has relationship with speed, a speed-dependent optimized filter should be considered to employ to track high mobility warship.

4.3.1 Input Model

The input model of FMP filter is the same model as illustrated in section 2.3.1.3. As parameters $a$ and $b$ are used to control target’s speed in Equations (2.29) and (2.30), in order to find the performance of filter influenced by speed, $a$ and $b$ are looped from 1 to 150 with step of 5.

4.3.2 Noise Addition

Since the measurements from the radar sensor contain errors, this was taken into account by adding a zero mean random white Gaussian noise with a standard deviation, $\sigma$ of 10 m to the true position sample. This error distribution in the observation is as shown in Figure (4.17) and Figure (4.18).
Figure 4.17 East-West error in the observation of speed-dependent $4^{th}$ order FMP filter to track high mobility target

Figure 4.18 North-South error in the observation of speed-dependent $4^{th}$ order FMP filter to track high mobility target
4.3.3 Determination of Optimal $\xi$

The filter’s weight constants $\alpha$, $\beta$, $\gamma$ and $\eta$ values depend on the value of the discounting factor $\xi$. Therefore, in order to obtain the optimal smoothing coefficients, optimization of the FMP filter focuses on adjusting the $\xi$ value experimentally through trial and error method.

The process of optimization begins by computation of the total prediction error which is the sum of the difference between the true trajectory and the predicted trajectory. The purpose of the optimization is to find the discounting factor that minimizes this error, and then to use this information to compute the optimal smoothing constants. This is, therefore, achieved by plotting a range of the discounting factor, which lies in the interval [0, 1] and with a step increase of 0.01. Thirty simulation tests are then carried out for each $\xi$ and the average is obtained. The resulting figure is as shown in Figure (4.19). According to this graph, the value of $\xi$ corresponding to the least residual error is 0.74. Meanwhile, total of smoothing tracking error which stands for the sum of difference between the true position and smoothed position is also used to optimize the filter. As shown in the Figure (4.20), the filter has the best smoothing performance when $\xi=0.74$. 
Figure 4.19 Sum of prediction tracking error of speed-dependent 4\textsuperscript{th} order FMP filter to track high mobility target.

Figure 4.20 Sum of smoothing tracking error of speed-dependent 4\textsuperscript{th} order FMP filter to track high mobility target.
4.3.4 Curve Fitting by Least Square Method

Curve fitting is a method of approximating discrete data with analytical expressions by using continuous curves to approximate the coordinate relationships represented by sets of discrete points. Least squares method is the most commonly used curve fitting method as it is more accurate and practical.

As mentioned above, the coefficients $a$ and $b$ in the input motion model are used to control the speed of target. In order to obtain enough data to do curve fitting, $a$ and $b$ are looped from 1 to 150 with step of 5. Since the data between $\xi$ and $v$ in Figure (4.21) are exponentially related, it is reasonable to assume an approximation of the form

$$\xi = c_2e^{c_1v}$$  \hfill (4.5)

Taking the logarithm of the both sides of Equation (4.5) yields a linear relation as

$$\ln \xi = c_1v + \ln c_2$$

$$= [v \quad 1][ \begin{array}{c} c_1 \\ \ln c_2 \end{array} ]$$  \hfill (4.6)

Applying the least squares method with given data points of 900, the estimate yields

$$\hat{x} = (A^TA)^{-1}A^T\xi$$  \hfill (4.7a)

where
\[
A = \begin{bmatrix} v_1 & 1 \\ v_2 & 1 \\ \vdots & \vdots \\ v_{900} & 1 \end{bmatrix} \in \mathbb{R}^{900 \times 2}, \quad \xi = \begin{bmatrix} \ln \xi_1 \\ \ln \xi_2 \\ \vdots \\ \ln \xi_{900} \end{bmatrix} \in \mathbb{R}^{900}, \quad \hat{z} = \begin{bmatrix} c_1 \\ \ln c_2 \end{bmatrix} \in \mathbb{R}^2
\] (4.7b)

Then \( c_1 \) and \( c_2 \) are can be obtained by

\[
c_1 = \hat{x}_1, \quad c_2 = e^{\hat{x}_2}
\] (4.8)

Using the obtained 900 sets of average speed and optimal \( \xi \), the curve fitting result can be obtained as Figure (4.21) and parameters calculation result is shown in Equation (4.9).

\[
\xi = -0.073 \cdot \ln(\nu) + 0.9641
\] (4.9)

**Figure 4.21** Curve fitting result of speed–dependent 4th order FMP filter to track high mobility target
4.3.5 Advantages of Speed-dependent FMP Filter

After comparing the tracking result of two filters, it is visibly clear that the speed-dependent fourth-order FMP filter has a higher performance than the fourth-order FMP filter with constant parameter. Quantitatively, the total prediction tracking error and total smoothing tracking error are used to evaluate the performance of two filters. As shown in the table below, the fourth-order FMP filter demonstrates the best performance when $\xi=0.74$ with the prediction tracking error amounting to 17,858.93 m. On the contrary, speed-dependent FMP filter depicts its optimal performance with prediction tracking error resulting to 16,372.76 m. The accuracy in tracking is therefore improved by 1486.17 m equivalent to 8.3% higher. Similarly, the smoothing accuracy is increased by 389.31 m by employing the speed-dependent FMP filter. Meanwhile, the standard deviation of prediction tracking error and smoothing tracking error of speed-dependent FMP filter are a little smaller than them of fourth-order FMP filter. It indicates that the stability of speed-dependent FMP filter is relatively higher than that of fourth-order FMP filter.

**Table 4.4** Performance comparison of speed-dependent fourth-order FMP filter and classic fourth-order FMP filter to track high mobility target

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error (m)</th>
<th>Smoothing tracking error (m)</th>
<th>Std. of prediction tracking error</th>
<th>Std. of smoothing tracking error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fourth-order FMP filter</td>
<td>17,858.93</td>
<td>10,226.53</td>
<td>9.09</td>
<td>5.24</td>
</tr>
<tr>
<td>Speed-dependent fourth order FMP filter</td>
<td>16,372.76</td>
<td>9,837.22</td>
<td>8.85</td>
<td>5.01</td>
</tr>
</tbody>
</table>
The speed-dependent FMP filter has another advantage that the parameters of filter are determined by the speed of target, and therefore there is no use to find the optimal tracking parameters before tracking, hence it can save time by this new method.

**Table 4.5** Performance comparison of speed-dependent third-order FMP filter and speed-dependent fourth-order FMP filter to track high mobility target

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error (m)</th>
<th>Smoothing tracking error (m)</th>
<th>Std. of prediction tracking error</th>
<th>Std. of smoothing tracking error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed-dependent third order FMP filter</td>
<td>18,632.96</td>
<td>10,496.26</td>
<td>9.45</td>
<td>5.32</td>
</tr>
<tr>
<td>Speed-dependent fourth order FMP filter</td>
<td>16,372.76</td>
<td>9,837.22</td>
<td>8.85</td>
<td>5.01</td>
</tr>
</tbody>
</table>

Table 4.5 shows the tracking performance comparison result of speed-dependent third-order FMP filter and speed-dependent fourth-order FMP filter to track high mobility target. The prediction tracking error of speed-dependent fourth-order FMP filter amounts to 16,372.76 m while the speed-dependent third-order FMP filter depicts its optimal performance with prediction tracking error resulting to 18,632.96 m. The accuracy in prediction is therefore improved by 2260.2 m equivalent to 12.13% higher. Similarly, the smoothing accuracy is increased by 659.04 m by employing the speed-dependent fourth-order FMP filter. Meanwhile, the standard deviation of prediction tracking error and smoothing tracking error of speed-dependent fourth-order FMP filter are smaller than them of speed-dependent third-order FMP filter which indicates that the stability of speed-dependent fourth-order FMP filter is relatively higher.
4.4 Speed-dependent Fourth-order FMP Filter to Track High Mobility Target under The Condition of Both Own Ship and Target in Motion

Section 4.3 discussed the speed–dependent fourth-order FMP filter to track high mobility warship with the condition that own ship is motionless. But it is much more common that own ship is also in motion. This Section explores the speed-dependent fourth-order FMP filter to track high mobility target under the condition of both own ship and target are in motion

4.4.1 Input Model of Own Ship and Target

The input model of own ship and target is the same model of Section 4.2. As parameters \(a, b, c\) and \(d\) are used to control target’s speed in Equations (4.3) - (4.6), in order to find the performance of filter influenced by speed, \(a, b, c\) and \(d\) are looped from 1 to 150 with step of 15. After 1000 times of simulation, 10000 sets of data of \(\xi\) and speed \(v\) will be obtained.

4.4.2 Noise Addition

The observed position is the output obtained from the radar measurements and therefore includes an error. In this study, the noisy observation was obtained by corrupting the true state with zero mean random white Gaussian noise of which standard deviation, \(\sigma\), is 10 m. Figure (4.22) and Figure (4.23) show the error distribution in the observation.
Figure 4.22 East-West error in the observation of speed-dependent 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.

Figure 4.23 North-South error in the observation of speed-dependent 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.
4.4.3 Determination of Optimal $\xi$

Figure (4.24) shows the enlarged view of true, observed, predicted and smoothed position trajectories lies in the interval [17800, 18600] in the X-axis. Figure (4.25) shows the prediction tracking error which stands for the difference between true position and predicted position while Figure (4.26) shows the smoothing tracking error which stands for the difference between true and smoothed position. Afterwards, set $\xi$ varies for 0 to 1 with step of 0.01 and calculated the total prediction tracking error and total smoothing tracking error of each step.

![Enlarged view of tracking result](image)

**Figure 4.24** Enlarged view of tracking result of speed-dependent 4$^{th}$ order FMP filter to track high mobility target under the condition of both own ship and target in motion.
Figure 4.25 Prediction tracking error of speed-dependent 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.

Figure 4.26 Smoothing tracking error of speed-dependent 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.
Figure 4.27 Sum of prediction tracking error and smoothing tracking error of speed-dependent 4th order FMP filter to track high mobility target under the condition of both own ship and target in motion.

Figure (4.27) shows the sum of total prediction tracking error and total smoothing tracking error when ξ value varies from 0 to 1. In this case the damping parameter, ξ was selected as 0.68 for prediction purpose.

4.4.4 Relationship between Speed and Optimal ξ

As mentioned above, curve fitting is used to find the relationship between the speed and optimal ξ. Setting coefficients $a, b, c, d$ in Equations (4.1) - (4.4) looped from 1 to 150 with step of 15, after plotting 10000 sets of $ξ - v$ points, it is clear to know that it exists a connection between $ξ$ and $v$. Since the data between $ξ$ and $v$ in Figure (4.28) are linearly related, it is reasonable to assume an approximation of a linear relation as

$$ξ = c_1 v^2 + c_2 v + c_3.$$  \hspace{1cm} (4.10)
Using the obtained 10000 sets of average speed and optimal $\xi$, the curve fitting result can be obtained as Figure (4.28).

\[
\begin{align*}
\xi &= 4 \times 10^{-5} \cdot v^2 - 0.006 \cdot v + 0.7674 \\
\end{align*}
\]

(4.11)

4.4.5 Advantages of Speed-dependent FMP Filter

After comparing the tracking result of two filters, it is visibly clear that
the speed-dependent fourth-order FMP filter has a higher performance than the classic fourth-order FMP filter with constant parameter. Quantitatively, the total prediction tracking error and total smoothing tracking error are used to evaluate the performance of two filters. As shown in Table below 4.6, the classic fourth-order FMP filter demonstrates the best performance when $\xi=0.50$ with the prediction tracking error amounting to 18,762.83 $m$. On the contrary, speed-dependent FMP filter depicts its optimal performance with total prediction tracking error resulting to 16,593.72 $m$. The accuracy in tracking is therefore improved by 2170.11 $m$ equivalent to 11.57\% higher. Similarly, the smoothing accuracy is increased by 409.28 $m$ by employing the speed-dependent FMP filter. Meanwhile, the standard deviation of prediction tracking error and smoothing tracking error of speed-dependent FMP filter are a little smaller than them of classic fourth-order FMP filter, it indicates that the stability of speed-dependent FMP filter is relatively higher than that of classic fourth-order FMP filter.

Table 4.6 Performance comparison of classic fourth-order FMP filter and speed-dependent fourth-order FMP filter to track high mobility target under the condition of own ship and target in motion

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error $(m)$</th>
<th>Smoothing tracking error $(m)$</th>
<th>Std. of prediction tracking error</th>
<th>Std. of smoothing tracking error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic fourth order FMP filter</td>
<td>18,762.83</td>
<td>10,274.56</td>
<td>10.16</td>
<td>5.31</td>
</tr>
<tr>
<td>Speed-dependent fourth order FMP filter</td>
<td>16,593.72</td>
<td>9,865.28</td>
<td>8.89</td>
<td>5.17</td>
</tr>
</tbody>
</table>

Table 4.7 shows the tracking performance comparison result of speed-dependent third-order FMP filter and speed-dependent fourth-order
FMP filter to track high mobility target under the condition of own ship and target in motion. The prediction tracking error of speed-dependent fourth-order FMP filter amounts to 16,593.72 m while the speed-dependent third-order FMP filter depicts its optimal performance with total prediction tracking error resulting to 21,059.89 m. The accuracy in prediction is therefore improved by 4466.17 m equivalent to 21.2% higher. Similarly, the smoothing accuracy is increased by 944.68 m by employing the speed-dependent fourth-order FMP filter. Meanwhile, the standard deviation of prediction tracking error and smoothing tracking error of speed-dependent fourth-order FMP filter are smaller than them of speed-dependent third-order FMP filter which indicates that the stability of speed-dependent fourth-order FMP filter is relatively higher.

### Table 4.7 Performance comparison of speed-dependent third-order FMP filter and speed-dependent fourth-order FMP filter to track high mobility target under the condition of own ship and target in motion

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Prediction tracking error (m)</th>
<th>Smoothing tracking error (m)</th>
<th>Std. of prediction tracking error</th>
<th>Std. of smoothing tracking error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed-dependent third order FMP filter</td>
<td>21,059.89</td>
<td>10,809.96</td>
<td>10.95</td>
<td>5.56</td>
</tr>
<tr>
<td>Speed-dependent fourth order FMP filter</td>
<td>16,593.72</td>
<td>9,865.28</td>
<td>8.89</td>
<td>5.17</td>
</tr>
</tbody>
</table>

### 4.5 Performance of Fourth-order FMP Filter to Track Random Motion Cases

In the front part of this Chapter, the motion model of all the cases is generated by combined trigonometric function with fixed parameters. This
results in the particularity of $\xi$ value. Therefore, it is necessary to check the performance of fourth-order FMP filter while the trajectories are different. This section, therefore, explores the performance of fourth-order FMP filter to track random motion cases.

### 4.5.1 Generation of Trajectory Function

The motion model of own ship and target is generated by multi-stage combined trigonometric function of which amplitude and wave length parameters are generated randomly.

1. Target motion

$$x_{t(n)} = c[c_{tx1}\sin(c_{tx2} \cdot t) + c_{tx3}\cos(c_{tx4} \cdot t) + c_{tx5}\sin(c_{tx6} \cdot t) + c_{tx7}\cos(c_{tx8} \cdot t) + c_{tx9}\sin(c_{tx10} \cdot t) + c_{tx11}\cos(c_{tx12} \cdot t) + c_{tx13}\sin(c_{tx14} \cdot t)] + 10n + 10000. \quad (4.12)$$

$$y_{t(n)} = d[c_{ty1}\sin(c_{ty2} \cdot t) + c_{ty3}\cos(c_{ty4} \cdot t) + c_{ty5}\sin(c_{ty6} \cdot t) + c_{ty7}\cos(c_{ty8} \cdot t)] + 3n + 10000. \quad (4.13)$$

where the parameters $c$ and $d$ are used to control the speed of target while $c_{tx1} \sim c_{tx14}$, $c_{ty1} \sim c_{ty8}$ are generated randomly to control the trajectory of target.

2. Own ship motion
\[ x_{o(n)} = a \left[ c_{ox1} \sin(c_{ox2} \cdot t) \right. \\
+ c_{ox3} \cos(c_{ox4} \cdot t) \\
+ c_{ox5} \sin(c_{ox6} \cdot t) \\
+ c_{ox7} \cos(c_{ox8} \cdot t) \\
+ c_{ox9} \sin(c_{ox10} \cdot t) \\
+ c_{ox11} \cos(c_{ox12} \cdot t) \\
+ \left. c_{ox13} \sin(c_{ox14} \cdot t) \right] + 10n, \]  

\[ (4.14) \]

\[ y_{o(n)} = b \left[ c_{oy1} \sin(c_{oy2} \cdot t) \right. \\
+ c_{oy3} \cos(c_{oy4} \cdot t) \\
+ c_{oy5} \sin(c_{oy6} \cdot t) \\
+ c_{oy7} \cos(c_{oy8} \cdot t) \\
+ \left. c_{oy13} \sin(c_{oy14} \cdot t) \right] + 3n. \]  

\[ (4.15) \]

where the parameters \( a \) and \( b \) are used to control the speed of target while \( c_{ox1} \sim c_{ox14} \), \( c_{oy1} \sim c_{oy8} \) are generated randomly to control the trajectory of target.

### 4.5.2 Determination of Parameters in Motion Model

As mentioned in Chapter 1, the majority of warships have speed upper limit of 30 knots. In order to simulate the tracking in a relatively verisimilar environment, the determination of parameters of motion models requires that the speed of target and own ship should lie in the interval of \((0, 60) m/s\). The parameters \( a, b, c, d \) are set as:

\[ a = 30, b = 50; c = 30, d = 50. \]  

\[ (4.16) \]

\[ c_{tx1} \sim c_{tx14}, c_{ty1} \sim c_{ty8}, c_{ox1} \sim c_{ox14}, c_{oy1} \sim c_{oy8} \] are determined by

\[ c = S \cdot U(0, 1) \]  

\[ (4.17) \]
where

\[ U(0, 1) \] is uniform function of interval of \([0, 1]\).

\( S \) is random scope, which indicates the expanded ratio of \( U \).

As the parameters are randomly generated for each simulation, the trajectory of original position and the curve of original true speed are different. Figure (4.29) – Figure (4.36) show four cases of data generation result, and it is visible that they have obviously difference in each case.
4.5.3 Simulation

The simulation can divided into 2 steps. The first step is to vary $\xi$ and plot the tracking error curve. The second step is to run 3000 times of first step, and statistics of the $\xi$ frequentness of each interval.

Step 1. $\xi$ loop from 0 to 1 with step of 0.01 and plot the residual tracing error curve.
Figure 4.37 Sum of prediction tracking error of random case 1

Figure (4.37) shows the residual tracking error of case 1 where trajectories of own ship and target are generated randomly. In this case $\xi$ is optimized as 0.51.

Step 2. Simulation of step 1 for 3000 loops.

In the second step, the simulation of step 1 is done for 3000 times, and 3000 sets of optimal $\xi$ can be obtained. Figure (4.38) shows the frequency distribution of 3000 sets of data. 422 sets of data in 3000 fall into the interval of [0.5, 0.51]. 99.5% of the data are lying in the interval [0.44, 0.6]. It indicates that the selection of parameters is relatively stable even though tracking various motion with different trajectories.
This Chapter discusses the performance of fourth-order FMP filter to track different cases. Speed-dependent fourth-order filter is also illustrated under the cases of only target in motion, and both target and own ship in motion. The last part tests the stability of filter with randomly generated motion parameters. The performance of fourth-order FMP filter can be summarized as follows:

1. The $\xi$ value is optimized by 0.74 when tracking high mobility target and own ship is static.

2. The $\xi$ value is optimized by 0.50 when tracking high mobility target and own ship is also in motion.

3. Speed-dependent fourth order FMP filter is developed to track the cases of only target in motion, and both own ship and target in motion. Comparisons of tracking result show that speed-dependent filter algorithms
have higher tracking accuracy and perform more stably than classic FMP filter.

(4) Fourth-order FMP filter and speed-dependent fourth-order FMP filter have higher accuracy and stability in predicting and smoothing than third-order FMP filter and speed-dependent third-order FMP filter.

(5) Combined trigonometric function motion case with randomly generated parameters is simulated and the optimized $\xi$ interval $[0.44, 0.6]$ is given in the last part.
Chapter 5 Conclusion

Tracking filters are an essential part of target tracking as they play a key role in tracking error reduction and making accurate estimations. Since the birth of the Kalman filter, many researchers have devoted themselves to improving the tracking accuracy and simplifying the algorithm of various filters. Despite this, tracking for a high mobility warship is still a complex challenge, and it has high maneuverability which leads to a quick change in speed and acceleration. This study discussed the performance of third and fourth-order FMP filter and recommends a speed-dependent algorithm to improve the tracking accuracy. Some main conclusions of this paper are drawn as follows.

Firstly, FMP filter is compared with other commonly used $\alpha-\beta-\gamma$ filter algorithms and Kalman filter. The superiorities are shown as follows:

(1). Compared with commonly used benedict-bordner filter algorithm and Gray&Murray filter algorithm, the FMP filter has the highest tracking performance in prediction and smoothing.

(2). FMP filter has another advantage that it is easy to implement due to its simplicity and low computational load.

(3). Even though the Kalman filter has a little higher tracking accuracy, the simulation results show that the Kalman filter appears to have a high rate of overshooting when tracking high mobility motion, hence it is relatively unstable compared with FMP filter.

(4). FMP filter is also easier to implementation and performs more competitively than Kanlam filter.

Secondly, the third-order FMP filter is optimized to track high mobility
target under the condition of own ship is motionless and in motion, and some conclusions can be drawn as follows:

(1). The $\xi$ value is optimized as 0.64 when tracking trigonometric function combined trajectory motion.

(2). The $\xi$ value is optimized as 0.56 when tracking trigonometric function combined trajectory motion under the high mobility motion of both own ship and target.

(3). Speed-dependent third-order FMP filter algorithm to track high mobility target is developed and it performs higher tracking accuracy and stability than classic FMP filter.

(4). Speed-dependent third-order FMP filter algorithm to track high mobility target under the condition of own ship and target in motion is developed and it performs higher tracking accuracy and stability than classic FMP filter.

Finally, the third-order FMP filter is extended to fourth-order FMP filter to track high mobility warship which has highly sudden changes in speed and acceleration. The conclusions are given as follows:

(1). The $\xi$ value is optimized by 0.74 when tracking high mobility target and own ship is static.

(2). The $\xi$ value is optimized by 0.50 when tracking high mobility target and own ship is also in motion.

(3). Speed-dependent fourth order FMP filter is developed to track the cases of only target in motion, and both own ship and target in motion. Comparisons of tracking result show that speed-dependent filter algorithms have higher tracking accuracy and perform more stability than classic FMP filter.
(4). Fourth-order FMP filter and speed-dependent fourth-order FMP filter have higher accuracy and stability in predicting and smoothing than third-order FMP filter and speed-dependent third-order FMP filter.

(5) Combined trigonometric function motion case with randomly generated parameters is simulated and the optimized $\xi$ interval $[0.44, 0.6]$ is given in the last part.

Optimization of FMP filter can not only improve the tracking accuracy for tracking high mobility warship, but also can save time to find gain parameters before tracking mission. However, some practical application problems such as controlling of glint noise or tracking multiple objects are still required to continue to research and study in the future.

In this study, a theoretical model was adopted to generate the motion dynamics of own ship and target. However, in a real environment the dynamics of own ship or target are calculated using radar measurements of range and bearing and are presented on the ARPA system. The authors therefore intend to apply the results of this study to a real situation in the near future whereby the range and angular bearings will be obtained in order to test the effectiveness of the proposed algorithm.
References


gain algorithm for maneuvering target tracking $\alpha-\beta-\gamma-\delta$ model. 


pp.370–376.


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