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Acoustic Field Due to An Arbitrary-Shaped Array and Propagation in Laterally Stratified Media Based on Global Matrix Method

임의배열 변환자로부터의 음파방사와 GMM을 사용한 적층매질에서의 전파모델

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ABSTRACT

Full wave solution to the 3-D radiation from a plane or volume array and propagation in a range independent wave guide are considered in this paper. The situation is encountered frequently when using directional source, i.e. array of sources in ocean environment. The three dimensional version of computer program "SAFARI" is extended for this purpose. The solution technique uses Global Matrix Method for a laterally stratified medium, which finds the full wave solution efficiently for the acoustic medium as well as elastic medium. Although the nature of this problem involves the Bessel function of higher order, which introduce the convergence problem, the problem can be greatly simplified by using superposition method for the solution to the individual sources. Numerical examples are presented to demonstrate the solution technique.

요 약

본 논문에서는 임의배열 변환자로 부터의 3차원 방사와, 그 음장의 적층매질에서 전파모델링을 다루었다. 이러한 문제는 실제 해양에서 방향성을 가지게 되는 변환자 배열을 사용할때 적용된다. 이 문제를 풀기위해서 3차원 모델의 "SAFARI"를 능률적으로 확장하였다. 여기에서 사용된 풀이방법은 GMM(Global Matrix Method)으로서, 해양저질층을 점성을 가진 탄성체로 모델링하여, 효과적으로 음장을 구하게 된다. 기존의 해법에서는, 방향성을 가진 경우 고차의 Bessel 함수로 인해 수치적인 수렴문제가 있었으나, 본 논문에서는 하나의 음원에 대한 완전해를 구함으로써, 변환자배열에 대한 간단한 변환을 하는 중첩모델을 사용하여 수치적인 수렴문제를 해결하였다. 제시된 풀이 방법을 사용하여 Gaussian Beam에 대한 해저면 반사음장과 전달음장에 대한 수치 예제를 제시하였다.

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I. Introduction

The propagation and radiation in the laterally stratified media has been a classical problem in the analysis and prediction of ocean acoustic pressure field. Among various methods available nowadays, Global Matrix Method[1][3] gives the full wave solution for acoustic waves as well as elastic waves of P, SV and SH, so as to treat

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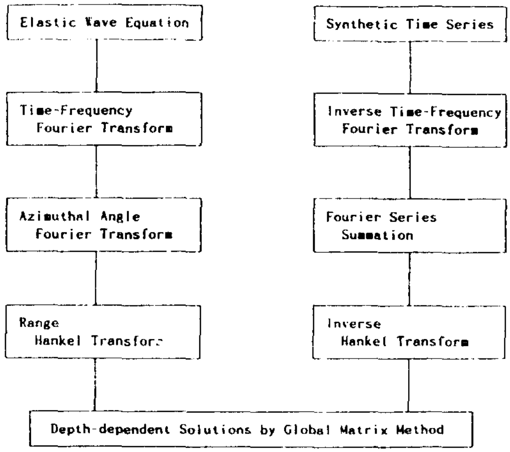
ocean bottom interactions. Another advantage comes from its ease of handling the directional sources such as couples[4], seismic sources[4,5] and line arrays[2][3][5].

The radiation from a horizontal array of sources has been treated in the paper by Schmidt and Glattetre[3]. However, the method is computationally intensive due to the higher orders of Bessel functions required to expand the range-direction field for each Fourier order in azimuthal angle. In this article, a superposition method, which eliminates Fourier expansion in azimuthal angle, is discussed and numerical examples are given with interpretations.

Ⅱ. Theory

The solution technique in a laterally stratified medium uses cylindrical coordinates. The advantage of using cylindrical coordinates in range independent environment is that only one integral transform in range direction is necessary since the azimuthal angle is in the form of Fourier summation. First, the equation of motion is depth-separated by Fourier transformation in the azimuthal angle and Hankel transform in range. The remaining ordinary differential equation in the depth coordinate with proper boundary conditions for horizontal interfaces yields a set of linear system of equation for each interface. The local matrix is properly combined to yield the global matrix, giving a full wave solution simultaneously for each azimuthal Fourier order, and for all layers. Once the depth-dependent solutions are found for each layer, the inverse Hankel transform is performed for each azimuthal Fourier order. Now, the frequency domain solution is found by summing the Fourier orders. The time domain solution is obtained by inverse Fourier transforming the solutions for each frequency. However, in this article, the transmission loss for single frequency, i.e. CW, is of interest.

The mathematical background for Global Matrix Method can be found in the cited



Figure(1) The Solution Procedure of Global Matrix Method

references[1][3][4]. Here, the method is briefly reviewed to show how the solution technique can be incorporated to give the full wave solution to the directional array radiation and propagation problem.

2.1 Wave Equation and Homogeneous Solution

The equation of motion for displacement in a homogeneous isotropic medium is

$$(\lambda+2\mu)\nabla(\nabla\cdot u)-\mu\nabla\times\nabla\times u+\rho F=\rho\frac{\partial^2 u}{\partial t^2} \quad (1)$$

where μ and λ are Lame's constants and u is displacement vector, ρ is density, and F is a body force vector. Introducing the force potentials, Φ, L, M, N, and displacement potentials ϕ, F, G, H, and subsequently Λ, ψ [6][4], the body force, F and displacement u are expressed as

$$F=(X,Y,Z)=\nabla\Phi+\nabla\times(L,M,L) \quad (2)$$

$$u=(u,v,w)=\nabla\phi+\nabla\times(F,G,H)$$
$$=\nabla\phi+\nabla\times\nabla\times(0,0,\Lambda)+\nabla\times(0,0,\psi) \quad (3)$$

The equation of motion reduces to homogeneous wave equation for scalar potentials

$$(\nabla^2+h^2)\phi=0$$
$$(\nabla^2+k^2)(\Lambda,\psi)=0 \quad (4)$$

where h and k denote wave numbers of compressional and shear waves, respectively. The potentials are expanded in Fourier series in the azimuthal angle θ, as

$$\phi(r,\theta,z)=\sum_{m=0}^{\infty}\phi^m(r,z)\begin{bmatrix}\cos m\theta\\ \sin m\theta\end{bmatrix}$$

$$\Lambda(r,\theta,z)=\sum_{m=0}^{\infty}\Lambda^m(r,z)\begin{bmatrix}\cos m\theta\\ \sin m\theta\end{bmatrix} \qquad (5)$$

$$\psi(r,\theta,z)=\sum_{m=0}^{\infty}\psi^m(r,z)\begin{bmatrix}\sin m\theta\\ -\cos m\theta\end{bmatrix}$$

where the displacement potentials for each azimuthal Fourier order can be written as

$$\phi^m(r,z)=\int_0^\infty[a_1^m(s)e^{-z\alpha(s)}+a_2^m(s)e^{z\alpha(s)}]\,sJ_m(rs)ds$$

$$\Lambda^m(r,z)=\int_0^\infty[b_1^m(s)e^{-z\beta(s)}+b_2^m(s)e^{z\beta(s)}]\,J_m(rs)ds \qquad (6)$$

$$\psi^m(r,z)=\int_0^\infty[c_1^m(s)e^{-z\beta(s)}+c_2^m(s)e^{z\beta(s)}]\,sJ_m(rs)ds$$

where s is horizontal wave number, and α and β represent vertical wave numbers of compressional and shear waves, respectively.

Substitution of the potentials into Eq(3) leads to the expression for the displacements, and the stresses can be found from constitutive relations. In the case of elastic medium interface, these 6 field parameters of displacements and stresses constitute the boundary conditions for each interface with 6 unknowns, which are

$$a_1^m(s),\ b_1^m(s),\ c_1^m(s),\ a_2^m(s),\ b_2^m(s),\ c_2^m(s) \qquad (7)$$

Next, the source term in the wave equation due to acoustic monopole is represented via force potentials.

2.2 Displacement Potential for Acoustic Monopole

The expression for the Green's function in The expression for the Green's function is transformed into cylindrical coordinates by Sommerfeld-Weyl Intergral [8]

$$\frac{e^{-ihR}}{R}=\int_0^\infty J_0(sr)e^{-\alpha|z|}\frac{s}{\alpha}ds \qquad (8)$$

Combining the above equation with Love-Stoke Formalism [6], the displacement potential can be shown to be

$$\Phi=\frac{M_0e^{iwt}}{4\pi\rho\omega^2}=\int_0^\infty\alpha h^2e^{-\alpha|z-z_s|}\frac{s}{\alpha}ds \qquad (9)$$

$$\Lambda=0$$

$$\psi=0$$

The corresponding displacements and stresses at the interface can be derived in the same way as the homogeneous solution. Detailed derivation can be found in the reference [4].

2.3 Global Matrix Method (GMM).

Once the field representation by homogeneous and inhomogeneous source terms is obtained, the problem is to find the unknown coefficients of the homogeneous solution in Eq(7). The boundary conditions of displacements and stresses constitute normally 6 boundary conditions at each interface, thus forming a set of local linear system of equations. Since the boundary conditions are to be satisfied at each boundary simultaneously, these sets of equations for each interface, when combined, form a global matrix. The unknown coefficients are found by solving the global matrix numerically, and the homogeneous solution for each layer is found by performing inverse Hankel transformation summing over Fourier orders for azimuthal angle.

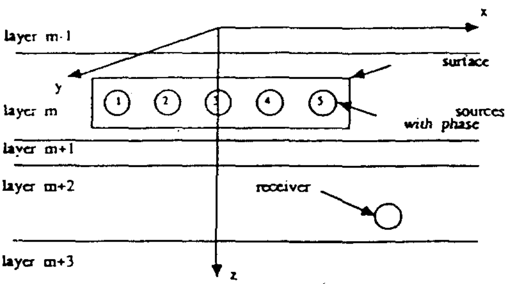
2.4 Field due to an array of sources-Superposition Method

The superposition method, literally, utilizes the superposition of radiated field by a single source placed on axis to represent the field caused by a source displaced from the z-axis (i.e. r=0 in cylindrical coord.). Figure(2) shows the horizontal array with sources off the z-axis. The field caused by this array can be represented by summation of the field in the receiver positions in Figure(3). For the horizontal displacements and shear stresses, the field parameters need to be transformed in the proper coordinate system, which can be either rectangular or cylindrical. Hence, the directivity caused by the horizontal line array can treat the horizontally or vertically polarized shear waves.

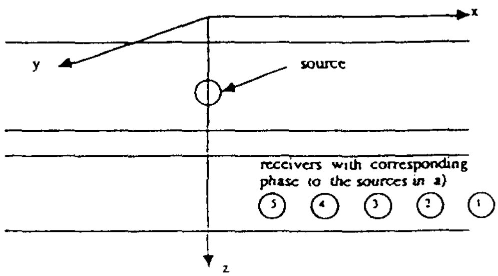
Ⅲ. Numerical Examples

3.1 Reflection from Ocean Bottom

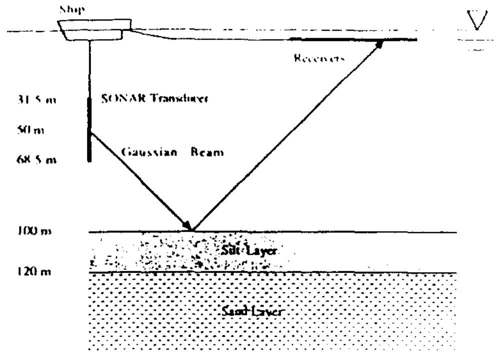
For the acoustic pressure field reflected from ocean bottom, two cases are discussed to demonstrate the applicability of the superposition method. The first example is the case of a vertical line array, which can be solved by 2-dimensional version of Global Matrix Method[1]. The layer consists of 3 layers. The uppermost layer is a fluid half space with sound speed 1500 m/sec. The second layer is the ocean bottom



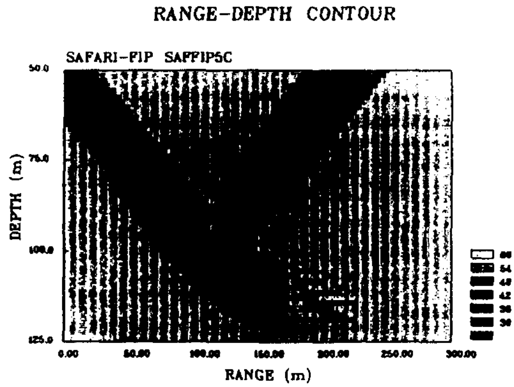
Figure(2) Array of horizontally distributed sources.



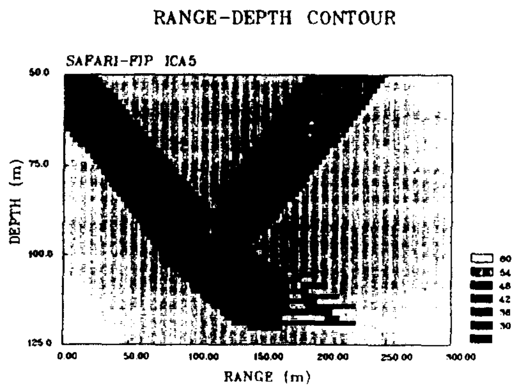
Figure(3) Reduced numerical Model of horizontally distributed sources.



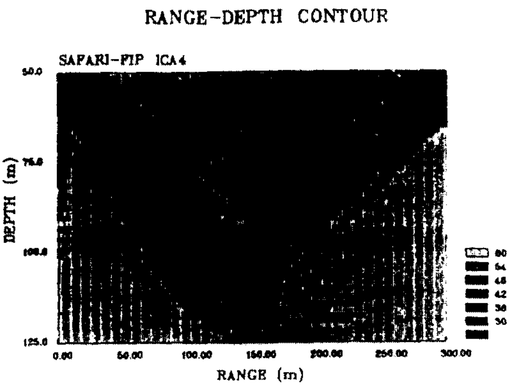
Figure(4) Sketch of geometry for the reflected field from ocean bottom.



Figure(5) Transmission loss by 2-dimensional version.



Figure(6) Transmission loss by Superposition Method.



Figure(7) Acoustic fileld due to horizontal line array.

with compressional wave speed of 1600 m/sec and shear wave speed of 400 m/sec, respectively. The third and last layer is the sub-bottom half space with compressional wave speed 1800 m/sec, and shear wave speed 600 m/sec, respectively. Figure(5) is the result by 2-dimensional version for the source array of 41 elements centered at 50 m steered to 25° downward and reflected from the ocean bottom. The y-axis is the depth axis from 50 m to 125 m. The x-axis is the range from 0 m to 300 m. The levels are intensity level of the acoustic field due to the array of sources normalized as 0 dB re lm and normalized as 0 dB re lm and 1μpa. Figure(5) is the result by 3-dimensional version using superposition method for the same geometric and source configurations. Figure(5) and Figure(6) show good agreement.

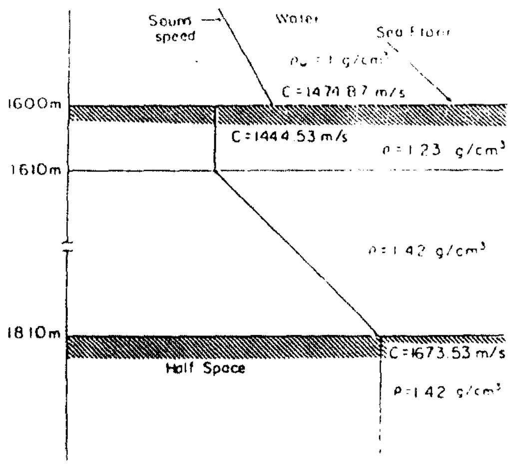
The second example is the case of a horizontal array which can only be treated in 3-dimensional version. Figure(7) shows the acoustic field from a horizontal array with 41 elements of 0.75 m apart steered to 65° (grazing angle 25°) in the vertical direction. This steering direction has been used to observe the same radiation direction as the previous example. As expected, the beam is broader than that of the first example, since the beam steering direction is toward the end-fire. The strong field in the near range is considered to be caused by interpolation error due to spatial sampling, which is required for field parameter transformation. This problem remains to be further studied and resolved.

3.2 Propagation in Stratified Ocean

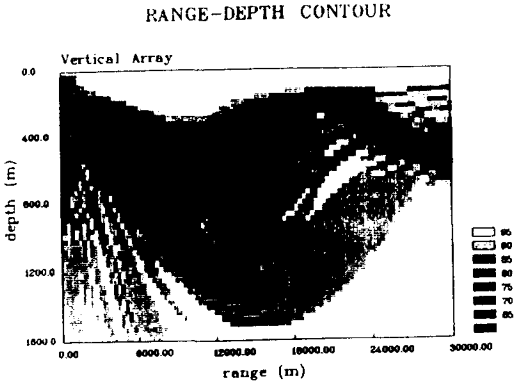
The acoustic feld for a relatively long range is treated in this section. The depth is 1600 m, and the sound velocity profile in the water is shown in Figure(10). The ocean bottom geometry and properties are shown in Figure(8). The absorption in the ocean bottom is 0.1 dB/km/Hz.

Figure(9) shows the acoustic field due to a vertical line array without steering. The source frequency is 100 Hz, and the total number of 21 sources are spaced 17.29 m apart, centered at the depth of 183 m, and weighted to radiate Gaussian beam pattern. The ray tracing diagram for the same environment, shown in Figure(10), agrees with the acoustic field shown in Figure(9). The vertical line array can be treated in 2 dimensional version of SAFARI[1]. Next, the pressure field caused by a slant array with 10° dip angle is shown in Figure(11), and the pressure field due to a downward vertical array with Gaussian beam steered 10° downward is shown in Figure(12). The same plots for 20° dip angle are shown in Figure(13) and Figure(14), respectively.

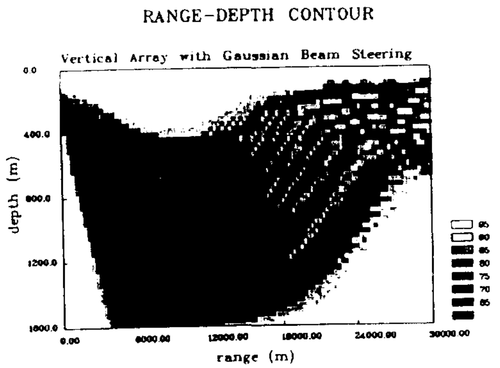
It is shown that the radiated field from slant arrays which can only be calculated by 3 dimensional version agrees well with the field from steered array radiaton, which confirms the applicability of the superposition method.



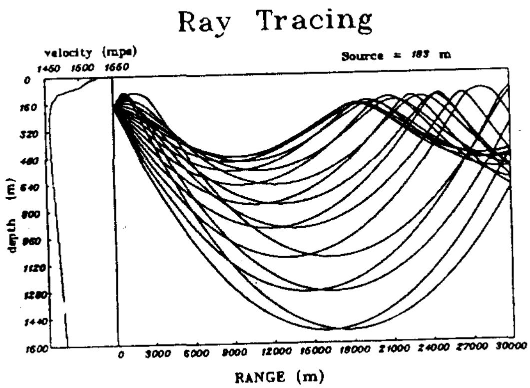
Figure(8) Ocean bottom properties.



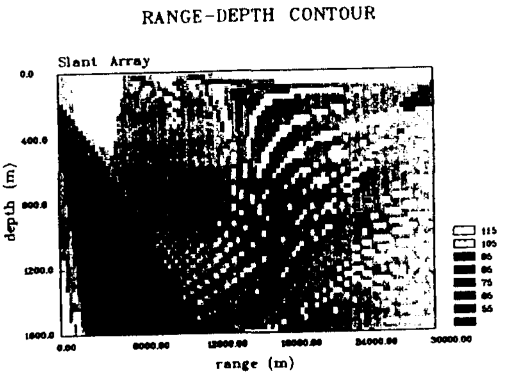
Figure(9) Acoustic field due to a vertical line array without steering(Gaussian beam pattern).



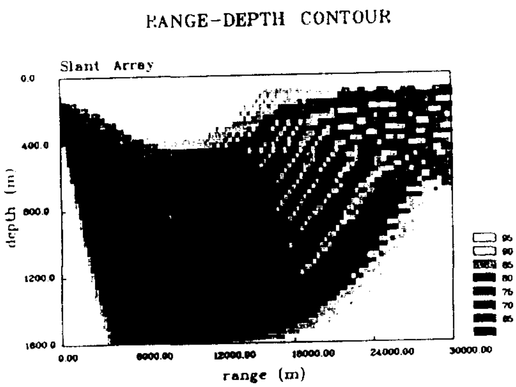
Figure(12) Acoustic field due to a vertical line array with Gaussian beam steering 10° downward.



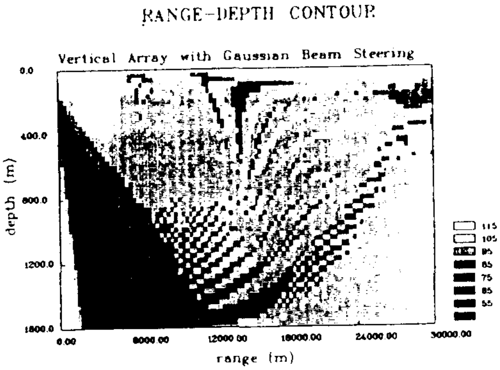
Figure(10) Ray tracing.



Figure(13) Acoustic field due to a 20° slant vertical line array.



Figure(11) Acoustic field due to a 10° slant vertical line array.



Figure(14) Acoustic field due to a vertical line array with Gaussian beam steering 20° downward.

Ⅳ. Summary

The "SAFARI" program based on Global Matrix Method has been extended to treat the arbitrary shaped array of acoustic monopoles by superposition. The superposition method is shown to be numerically efficient without convergence problem, while the previous field representation by Fourier series in azimuthal angle involves higher orders of Bessel function causing numerical convergence problem for highly directional sources. Numerical examples show that the method describes the 3-dimensional characteristics of the acoustic field accordingly.

Ackowledgements

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