Agency with Minimum Output Requirement

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Abstract

This article focuses on the probable influence of minimum requirement terms on principal-agent problems with information asymmetry. They are indeed common in real-world situations and affect the power of incentive schemes that the principal suggests to agents *ex ante*. In particular, they are significant to the cases in which the final output is dependent only upon the best one; high minimum requirement levels could restrict the power of incentive schemes to be asymmetric. What matters more is the fact that if they are set excessively high, a project that would be profitable may be abandoned resulting in lowering social welfare.

1 Introduction

During the past two decades, the theory of principal and agent has led economists and policy makers to the reconsideration on the thorough understanding of the standard theory of perfect competition under complete markets and, more importantly, to the resulting realization that the neoclassical paradigm is insufficient to accommodate a number of important economic phenomena. It is the major alternative to models of imperfect competition. Contingent commodity trades of the Arrow-Debrue type are a typical examples. General tradition of analysis in the literature follows under the standard assumption of aggregate and/or additive objective of the principal.

One of the most important variations in the interpretation of the results from agency theory, however, is that often the principal's objective is not a simple function of agent's *outcomes*. Prototypical example is that of research and development. If the principal is a benevolent social planner, his preferences regarding the result of individual R&D ought to be ill-behaved. Optimal incentive schemes of this study are based on such situations.

Furthermore, considered is the case such that a certain kind of strict minimum requirement terms are imposed. Minimum requirement terms play an important part on determining validity of a project; if they were not satisfied ex ante, the principal would have no motivation to keep the project activated. They are frequently observed in reality and essential in many agency relationships such as procurement, advertising, and object-oriented programs; An electric power company knows the least total capacity of generators in operation

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sufficient to serve its customers safely without disastrous blackout. (The sum of outputs is critical.) Or, R&D teams of an automotive company may be ordered to improve their automobiles to meet reinforced legal safety standards, which function as a minimum requirement. (Only the best among outputs is critical.)

Of examples suggested above, what are being mainly considered here are cases such as the latter, though a case such as the former is also covered. There are a few previous studies that deserve mentioning in this context.

A study by Olsen (1993) focuses on the R&D regulation problems raised by two particular characteristics of research and development activities: first, that it is normally desirable that several units pursue the same goal, and second, that only the best of the final products made by these units is worthwhile to society. Therefore, a substitution effect comes to exist among agents' outputs. [1] Those ideas were imported into this article.

Levitt(1995) also points out the difference of incentive schemes adopted when only the best output matters (an R&D activity for a realistic example) from that of standard principal-agent models. [2] The most important contribution of his paper is the fact that the optimal incentive scheme may differ across agents even when identical agents perform identical tasks. And it needs noticing that employing more than one agent for a project is supported by sampling effect. [3]

2 Theory of Principal and Agents: An Overview

Agency relationships are ubiquitous in economic life. Wherever there are gains to specialization there is likely to arise a relationship in which agents act on behalf of a principal, because of comparative advantage. The economic value of decision making made on behalf of someone else would easily seem to match the value of individual consumption decisions. In this light the attention paid to agency problems has been relatively slight.

If agents could costlessly be induced to internalize the principal's objectives, there would be little reason to consider agency problems. Things become interesting only when objectives cannot be automatically aligned. Then, what is it that prevents inexpensive alignment? The most plausible and commonly offered reason is *asymmetric information*, which of course ties closely to the source of agency: returns to specialization. The sincerity of a worker's labor input is often hard to verify, leading to problems with shirking. Informational expertise permits managers to pursue goals of their own such as enhanced social status or improved career opportunities. Private information about individual characteristics causes problems for the government in collecting taxes.

Thus, underlying each agency model is an *incentive* problems caused by some form of asymmetric information. It is common to distinguish models based on the particular information asymmetry involved. A generally accepted typical taxonomy is as follows.

Moral Hazard All models which assume symmetric information at the timing of contracting, but with some kind of *post-contractual* information asymmetry, are models of moral hazard. A classic example is fire insurance,

[1] More resources allocated to R&D by one agent reduces the social marginal value of the other firms' R&D activity.

[2] He regards a standard model as one in which the final output is the sum of outputs.

[3] Many draws from a distribution are better for the principal than a single draw.

where the insuree may or may not exhibit sufficient care while storing flammable materials.

Adverse Selection All models in which the agent has *pre-contractual* information are models of adverse selection. A classic example is life insurance, where the insuree may know things about the state of her health that are unknown to the insurer.

In some situations, models of moral hazard are further classified into the case where the agent takes unobservable actions, and the case where his actions (but not the contingencies under which they were taken) may be observed. Arrow (1985) has suggested the informative names "Hidden Action Model" and "Hidden Information Model" for these two subcategories. The worker supplying unobservable effort is the typical hidden action case, while the expert manager making observable investment decisions leads to a typical hidden information model. What is more important, however, than these categorization is the precise understanding of the "rules of the game" — *who knows what when, who does what when* (Kreps, 1990).

The general objective of an agency analysis is to characterize the optimal organization response to the incentive problem. Typically, the analysis delivers a second-best reward structure for the agent, based on information that can be included in the contract. Characterizing the optimal incentive scheme is important but not the prime economic purpose. What is more interesting is the allocational distortions that come with the incentive solutions. Although one could often design incentive schemes that induce the agent to behave as if no information asymmetry were present, that is rarely second-best. Instead, some of the costs of the information asymmetry are borne by distortions in decision rules, task assignments, and other costly institutional arrangements. This is what gives the theory of principal and agents its main economic content.

3 Optimal Incentive Schemes under Minimum Requirement

A profit-maximizing, risk-neutral principal wants to have a projects accomplished by agents. Its payoff depends on the total wage bill given to agents and the final output yielded by them. Explaining the term of 'final output', when the sum of each agents' outputs is significant to the principal, the final output means it, for instance. Similarly, either the average or only the best one can be a remarkable candidate for the final output under some relevant conditions respectively. Principal's utility would increase with one additional unit of the final output and decrease with one unit of wages raised by one unit respectively. Most of all, there is set a severe minimum requirement term for the final output to satisfy necessarily. If the expected final output were below a designated minimum requirement level, it would have to abandon the project.

Each agent employed works alone so that consideration of the free-rider problem arising in teams is eliminated (Holmström, 1982). Divisibility of its output is not so critical on the premise that the principal is prohibited from combining the partial elements of multiple agents' output that is necessary or attractive.

An agent's output is jointly determined by its effort level and a stochastic component, which causes a certain moral hazard problem with the fact that the effort level is not observable by the principal. But, observable by the principal and all other agents is an agent's output x_i which is determined as $x_i = e_i + \epsilon_i + \eta$, where the effort level $e_i \in [0, \infty)$. [4] That is to say, no agent will be able to act destructively against the principal.

ϵ_i is an idiosyncratic shock and η is a common shock across all agents. The agent-specific shock is uncorrelated with the common shock. Shocks are assumed to be individually and independently distributed across all agents with mean zero. The agents observe the sum of the shock, but they cannot distinguish between the idiosyncratic and common component of the shock. Agents are unable to coordinate their efforts, ruling out consideration of collusion. They also make their efforts not concerned with anticipated behavior of other agents.

Agents are assumed to be risk-neutral and identical in every single respect. Therefore, adverse selection does not exist in the model. They have identical reservation utilities u_0, which the principal will have to compensate them for. Effort level e_i gives an agent quadratic disutility, $e_i^2/2$.

One unit of wages amounts to one unit of utility for every agent. Moreover, it is assumed that optimal incentive schemes are linear in the principal's observables for the reason such as practical implementability. [5]

The principal's problem is analyzed in three cases, which are n-agent sum-critical case, 2-agent best-critical case and confined n-agent best-critical case.

3.1 N-agent Sum-critical Case

The principal's problem can be defined as follows:

(P1)

$$\begin{aligned} \max_{\alpha_1,\ldots,\alpha_n,\beta_1,\ldots,\beta_n,\gamma_1,\ldots,\gamma_n} \quad & E\left(\textstyle\sum_{i=1}^n x_i\right) - E\left(\textstyle\sum_{i=1}^n W_i\right) \\ \text{s. t.} \quad & e_i^* \in \arg\max E\left(U_i(e_i)\right) && \text{(IC)} \\ & E\left(W_i\right) - \tfrac{e_i^2}{2} \geq u_0 && \text{(IR)} \\ & E\left(\textstyle\sum_{i=1}^n x_i\right) \geq k && \text{(Min. Req.)} \end{aligned}$$

where for $i = 1, \ldots, n$, the wages paid to agent i is

$$W_i = \alpha_i + \beta_i x_i + \gamma_i \sum_{j \neq i} x_j$$

and its utility is given by

$$U_i(W_i, e_i) = W_i - \frac{e_i^2}{2}$$

Agent i's output is $x_i = e_i + \epsilon_i + \eta$ as described above.

The principal is assumed to be a private firm so that it maximizes its own profit minus wages to agents. [6] It will be the same at all of the following cases.

[4] The output x_i may imply quantity of produced goods, length of time period took to complete the first prototype, or market value of newly synthesized material in the real world.

[5] See Sappington (1982), p.356

[6] For more about the difference between a principal as a 'planner' and as a private firm, see Laffont and Tirole, p.618

Solving (IC) constraint with

$$E(U_i) = \alpha_i + \beta_i e_i + \gamma_i \sum_{j \neq i} e_j - \frac{e_i^2}{2}$$

agent i's optimal choice of action is given by

$$e_i^* = \beta_i^*$$

It is due to the concavity of the agent's problem which makes it necessary and sufficient to take first-order conditions of (IC) constraint. And the principal would set W_i to get the (IR) constraint to bind, i.e.,

$$E(W_i) = u_0 + \frac{e_i^2}{2}$$

Then the problem (P1) becomes

$$\begin{aligned} \max_{\beta_1,\ldots,\beta_n} \quad & \sum_{i=1}^{n} \beta_i - \frac{1}{2}\sum_{i=1}^{n} \beta_i^2 - nu_0 \\ = \quad & n(\frac{1}{2} - u_0) - \frac{1}{2}\sum_{i=1}^{n}(\beta_i - 1)^2 \\ \text{s. t.} \quad & E\left(\sum_{i=1}^{n} x_i\right) \geq k \end{aligned}$$

In the case that the reservation utility $u_0 \leq 1/2$, i.e., the project is thought of as a profitable one, the principal will have each agent exert the effort level $e_i^* = \beta_i^* = 1$ and take an expected net profit of $n(1/2 - u_0)$. Naturally, it would want to employ as many agents as possible. And, the minimum requirement constraint is satisfied for at least $n = [k+1]$ agents automatically. An agent's expected wages is $u_0 + 1/2$ and

$$\alpha_i + (n-1)\gamma_i = u_0 - \frac{1}{2}$$

Needless to say, this result raises a problem of arbitrariness. Notice that the assumption on infinite supply of identical agents is not of reality. The number of employed agents will not diverge to infinity but be constrained by either the finite number of agents which are capable of the project (finiteness of n) or the break-even point which is met because the more agents are employed, the higher marginal cost of hiring (heterogeneous agents).

If $u_0 \geq 1/2$, the value of the objective function always turns out to be negative at the very most. Then the principal would give up the project by offering unacceptable wages to agents, for example. In addition, those specified above are the first-best optimum. It is known that an first-best optima is attained under the setting such as risk-neutral principal and agents with moral hazard.

3.2 2-Agent Best-Critical Case

Now, consider cases where the critical final output is a single best output. The principal's problem here becomes

(P2)

$$\begin{array}{rcl} \max_{\alpha_1,\alpha_2,\beta_1,\beta_2,\gamma_1,\gamma_2} & E\left(\max\{x_1,x_2\}\right)-E\left(W_1+W_2\right) & \\ \text{s. t.} & e_i^* \in \arg\max E\left(U_i(e_i)\right) & \text{(IC)} \\ & E\left(W_i\right)-\frac{e_i^2}{2} \geq u_0 & \text{(IR)} \\ & E\left(\max\{x_1,x_2\}\right) \geq k & \text{(Min. Req.)} \end{array}$$

where for $i=1,2$,

$$\begin{aligned} W_i &= \alpha_i+\beta_i x_i+\gamma_i x_j \\ x_i &= e_i+\epsilon_i+\eta \\ U_i &= W_i-\frac{e_i^2}{2} \end{aligned}$$

Similarly as in the previous case, problem (P2) can be transformed into;

$$\begin{array}{rl} \max_{\beta_1,\beta_2} & E\left(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\}\right)-\frac{1}{2}\left(\beta_1^2+\beta_2^2\right)-2u_0 \\ \text{s. t.} & E\left(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\}\right) \geq k \end{array} \quad (1)$$

As to $E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\}$, the following identity always holds: [7]

$$\begin{aligned} P(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\} \leq z) &= P(\beta_1+\epsilon_1 \leq z)P(\beta_2+\epsilon_2 \leq z) \\ &= P(\epsilon_1 \leq z-\beta_1)P(\epsilon_2 \leq z-\beta_2) \end{aligned} \quad (2)$$

Let the probability density function of ϵ_i be $f(\cdot)$. Then,

$$E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\}) = \int_{-\infty}^{\infty} zf(z)dz \quad (3)$$

where

$$f(z)=\frac{\partial}{\partial z}P(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\} \leq z) \quad (4)$$

To observe and examine the problem in detail, a tangible shapes of

$$E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\})$$

is essential. Thus, idiosyncratic shocks ϵ_i are supposed to assume one of two zero-mean distributions — uniform distribution and double exponential distribution. They are chosen for their properties such as zero-mean and mathematical tractability. Premised is that the minimum requirement constraint is satisfied from now on, if not specified.

Finally, by the same token as in the sum-critical case, the first-best optima are reached in subsequent problems.

3.2.1 Shocks from Uniform Distribution

Assume that $\beta_1 \geq \beta_2$ hereafter for they are identical agents. Assume also that ϵ_i's are uniformly distributed along the interval $(-\delta,\delta)$ where $\delta>0$. By intuition, it is straightforward that no matter who the best output has come

[7] A simple order statistics

from, it should be larger than $\beta_1-\delta$. So, the problem should be categorized into two parts according to the existence of an intersection between the supports of two agents' outputs.

First, if there is no intersection at all, i.e., $\beta_1-\delta>\beta_2+\delta$, only agent 1's output matters. Therefore,

$$E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\})=E(\beta_1+\epsilon_1)=\beta_1$$

If the minimum requirement is satisfied, the objective function 1 changes into

$$h_{\mathrm{A}}(\beta_1,\beta_2)=-\frac{(\beta_1-1)^2}{2}-\frac{\beta_2^2}{2}+\frac{1}{2}-2u_0$$

Hence, the optimum effort level of two agents is $\beta_1^*=e_1^*=1,\beta_2^*=e_2^*=0$ and the expected value of the objective function is

$$h_{\mathrm{A}}^*=\frac{1}{2}-2u_0$$

Note that from the viewpoint of the principal it is inferior to what could be got with a single agent (Refer to n-agent sum maximizing case with $n=1$. Because small δ means negligible sampling effect, it is natural that only one agent should make efforts and contribute to the principal's utility. In effect, the principal would not employ multiple agents as long as it could not expect any significant δ. The principal pays expected wages of $(u_0+1/2)$ to agent 1 and just the reservation utility u_0 to agent 2. [8] Then, $\alpha_1=(u_0-1/2)$ and $\alpha_2+\gamma_2=u_0$. To make conclusion it is not only necessary but also sufficient for this asymmetric scheme to be optimal that $\delta\le 1/2$.

When $\beta_1-\delta\le\beta_2+\delta$, however, the story goes very differently. Let $F(\cdot)$ denote the cumulative distribution function of ϵ_i. From the equations (2) and (4),

$$F(z)=\begin{cases}0 & \text{if} \quad z\le\beta_1-\delta\\ \frac{(z-(\beta_1-\delta))(z-(\beta_2-\delta))}{4\delta^2} & \text{if} \quad \beta_1-\delta\le z\le\beta_2+\delta\\ \frac{z-(\beta_1-\delta)}{2\delta} & \text{if} \quad \beta_2+\delta\le z\le\beta_1+\delta\\ 1 & \text{if} \quad \beta_1+\delta\le z\end{cases}$$

Now, the probability density of ϵ_i, $f(\cdot)$ can be obtained by differentiating $F(\cdot)$ as follows.

$$f(z)=\begin{cases}\frac{2z+2\delta-(\beta_1+\beta_2)}{4\delta} & \text{if} \quad \beta_1-\delta\le z\le\beta_2+\delta\\ \frac{1}{2\delta} & \text{if} \quad \beta_2+\delta\le z\le\beta_1+\delta\\ 0 & \text{otherwise}\end{cases}$$

With this $f(z)$ applied to the equation (3),

$$\begin{aligned}E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\}) &= \int_{\beta_1-\delta}^{\beta_2+\delta}\left(\frac{(2z+2\delta-(\beta_1+\beta_2))z}{4\delta}\right)dz\\ &+\int_{\beta_2+\delta}^{\beta_1+\delta}\left(\frac{1}{2\delta}\right)dz \qquad (5)\end{aligned}$$

[8] Here u_0 is analogous to opportunity cost which should be made up.

Because of its outrageously complicated form, an in-depth analysis on the expected final output is indeed impossible. And yet, some available implication has come out as far as the attention is restricted to symmetric β_i's. Namely,

$$E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\})=\beta_1+\frac{\delta}{3}$$

from (5) if $\beta_1=\beta_2$. And the objective function (1) becomes

$$h_S(\beta_1,\beta_2)=-\left(\beta_1-\frac{1}{2}\right)^2+\left(\frac{1}{4}+\frac{\delta}{3}\right)-2u_0$$

Hence, optimal symmetric incentive powers are $\beta_1^*=\beta_2^*=1/2$ and principal's maximum profit is expected to be

$$h_S^*=\frac{1}{4}+\frac{\delta}{3}2u_0$$

which is more definitely attractive than h_A^* as long as $\delta\geq 3/4$. Then,

$$E(W_1)=E(W_2)=u_0+\frac{1}{8}$$

and,

$$\alpha_1+\frac{\gamma_1}{2}=\alpha_1+\frac{\gamma_1}{2}=u_0-\frac{1}{8}$$

As is seen, δ is plays a significant role in determining the shape of incentive schemes and principal's expected profit here.

3.2.2 Shocks from Double Exponential Distribution

Now assume that ϵ_i's are independent and follows *double exponential* distribution. The expected final output $E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\})$ can be obtained in the same manner as in the case of uniform distribution. First, cumulative distribution function of maximum output $G(\cdot)$ can be shown as follows.

$$\begin{aligned}G(z)&=\frac{1}{4}\int_{-\infty}^{z-\beta_1}e^{-|y|}dy\int_{-\infty}^{z-\beta_2}e^{-|y|}dy\\&=\begin{cases}\frac{1}{4}e^{2z-(\beta_1+\beta_2)} & \text{if } z\leq\beta_2\\ \frac{1}{4}e^{z-\beta_1}(2-e^{-z+\beta_2}) & \text{if } \beta_2\leq z\leq\beta_1\\ \frac{1}{4}(2-e^{-z+\beta_1})(2-e^{-z+\beta_2}) & \text{if } \beta_1\leq z\end{cases}\end{aligned}$$

Then, upon differentiation, corresponding probability density, $g(\cdot)$ becomes as follows.

$$g(z)=\begin{cases}\frac{1}{2}e^{2z-(\beta_1+\beta_2)} & \text{if} & z\leq\beta_2\\ \frac{1}{2}e^{z-\beta_1} & \text{if} & \beta_2\leq z\leq\beta_1\\ \frac{1}{2}\left(e^{\beta_1}+e^{\beta_2}\right)e^{-z}-\frac{1}{2}e^{\beta_1+\beta_2-2z} & \text{if} & \beta_1\leq z\\ 0 & \text{otherwise} & \end{cases}\qquad(6)$$

From this, after some tedious algebra, it can be shown that

$$E(\max\{\beta_1+\epsilon_1,\beta_2+\epsilon_2\})=\beta_1+\frac{1}{4}(\beta_1-\beta_2+3)e^{\beta_2-\beta_1}$$

Then the principal's profit function, $\kappa(\cdot)$ becomes as follows.

$$\kappa(\beta_1, \beta_2) = \beta_1 + (\beta_1 - \beta_2 + 3)e^{\beta_2 - \beta_1} - \frac{\beta_1^2 + \beta_2^2}{2} - 2u_0$$

Note that the profit function $\kappa(\cdot)$ is strictly concave which assures the (unique) global maximum. Now, the problem (P1) in this case can be rewritten as

$$\begin{aligned} \max_{\beta_1, \beta_2} \quad & \kappa(\beta_1, \beta_2) \\ \text{s. t.} \quad & \beta_1 + \tfrac{1}{4}e^{\beta_2 - \beta_1}(\beta_1 - \beta_2 + 3) \geq k \end{aligned}$$

Taking first-order conditions yield

$$\begin{aligned} \frac{\partial \kappa}{\partial \beta_1} &= -\frac{1}{4}e^{\beta_2 - \beta_1}(\beta_1 - \beta_2 + 2) + 1 - \beta_1 = 0 \\ \frac{\partial \kappa}{\partial \beta_2} &= \frac{1}{4}e^{\beta_2 - \beta_1}(\beta_1 - \beta_2 + 2) - \beta_2 = 0 \end{aligned}$$

After rearranging terms, one can easily verify that the optimal choice of β_1 and β_2 must satisfy

$$\begin{aligned} \beta_1^* &= 1 - \frac{1}{4}e^{\beta_2 - \beta_1}(\beta_1 - \beta_2 + 2) \\ \beta_2^* &= \frac{1}{4}e^{\beta_2 - \beta_1}(\beta_1 - \beta_2 + 2) \end{aligned}$$

Note that the preceding conditions are exactly equivalent to the following conditions, which can provide a more intuitive interpretation.

$$\begin{aligned} \beta_1^* + \beta_2^* &= 1 \\ \beta_1^* - \beta_2^* &= 1 - \frac{1}{2}e^{\beta_2^* - \beta_1^*}(2 - (\beta_2^* - \beta_1^*)) \end{aligned}$$

By means of these conditions the objective function $\kappa(\beta_1, \beta_2)$ can be transformed into a function of one variable. Let $x = \beta_2 - \beta_1$ and note that $\beta_1 = (1-x)/2$ and $\beta_2 = (1+x)/2$ where $-1 \leq x \leq 0$. Now, the objective function becomes as follows.

$$\begin{aligned} 4\kappa(x) &= e^x(3-x) - (x+1)^2 + 2 - 8u_0 \\ 4\kappa'(x) &= e^x(2-x) - 2(x+1) \end{aligned}$$

Note that $\kappa'(0) = 0$. This implies that $\kappa(x)$ attains its maximum value $1 - 2u_0$ at $x = 0$.

Meanwhile, expected final output $\mathcal{O}(\cdot) = E(\max\{\beta_1 + \epsilon_1, \beta_2 + \epsilon_2\})$ can be expressed as a function of x as follows.

$$4\mathcal{O}(x) = e^x(3-x) + 2(1-x)$$

with

$$\begin{aligned} 4\mathcal{O}'(x) &= e^x(2-x) - 2 \\ 4\mathcal{O}''(x) &= e^x(1-x) \end{aligned}$$

Examined within the domain of x through its first-order and second-order derivatives, it is found to be a monotonely decreasing function which has its minimum of 1.25 at $x = 0$ and maximum of $e^{-1} + 1$ at $x = -1$.

Thus, if the minimum requirement level is below the minimum value of the expected final output, the principal will be able to reach the global optimum with value(profit) $1 - 2u_0$. Otherwise, only a restricted maximum can be attainable. Let κ^* and $\mathcal{O}^*$ be the optimal values of $\kappa(\cdot)$ and $\mathcal{O}(\cdot)$, respectively. Then, there can be two cases;

1. $\kappa^* = 1 - 2u_0$ and $\mathcal{O}^* = 1.25$ if $k \leq 1.25$
2. $\kappa^* = \kappa(\mathcal{O}^{-1}(k))$ and $\mathcal{O}^* = k$ if $1.25 \leq k \leq 1 + e^{-1}$

The results above means that minimum requirement parameter k has notable effect on the value of the objective function and powers of wage schemes. From the viewpoint of wage schemes, symmetric powers, $\beta_1^* = \beta_2^* = 1/2$, are preferred if $k \leq 1.25$. The principal's expected profit amounts to $1 - 2u_0$ and expected wages of $u_0 + \frac{1}{8}$ are paid to each agent. otherwise, asymmetric powers,

$$\begin{aligned} \beta_1^* &= \frac{1 - \mathcal{O}^{-1}(k)}{2} \\ \beta_2^* &= \frac{1 + \mathcal{O}^{-1}(k)}{2} \end{aligned}$$

are preferred by the principal. Wages and characteristics of the optimal wage schemes can be found without difficulty. Finally, to compare those cases let social welfare be defined as the sum of the principal's and agents' utilities. For an agent always get the utility of u_0 with individual rationalty constraints binding, and $\kappa(\mathcal{O}^{-1}(k)) \leq 1 - 2u_0$, it is observed that social welfare is decreased by the existence of the minimum requirement in latter case.

3.3 N-agent Best-Critical Case

In this section, we will confine ourselves to shocks from double exponential distribution. Further, for the sake of simplicity, incentive powers are constrained to be symmetric. Then the principal's problem becomes as follows.
(P3)

$$\begin{aligned} \max_{\alpha_1,\ldots,\alpha_n,\beta_1,\ldots,\beta_n,\gamma_1,\ldots,\gamma_n} \quad & E(\max\{x_1,\ldots,x_n\}) - E\left(\textstyle\sum_{i=1}^n W_i\right) \\ \text{s. t.} \quad & e_i^* \in \arg\max E\left(U_i(e_i)\right) & \text{(IC)} \\ & E(W_i) - \tfrac{e_i^2}{2} \geq u_0 \ \forall i & \text{(IR)} \\ & E(\max\{x_1,\ldots,x_n\}) \geq k & \text{(Min. Req.)} \end{aligned}$$

where for $i = 1, \ldots, n$

$$W_i = \alpha_i + \beta_i x_i + \gamma_i \max\{x_1, \ldots, x_i - 1, x_i + 1, \ldots, x_n\}$$

$$x_i = e_i + \epsilon_i + \eta$$

$$U_i = W_i - \frac{e_i^2}{2}$$

In addition,

$$\beta_1 = \beta_2 = \ldots = \beta_n$$

as was supposed to be. Through similar procedure applied in the previous section, the problem (P3) becomes as follows.

$$\max_{\beta_1,\ldots,\beta_n} \quad E(\max\{\beta_1+\epsilon_1,\ldots,\beta_n+\epsilon_n\}) - \tfrac{1}{2}\textstyle\sum_{i=1}^{n}\beta_i^2 - nu_0 \tag{7}$$
$$\text{s. t.} \quad E(\max\{\beta_1+\epsilon_1,\ldots,\beta_n+\epsilon_n\}) \geq k$$

As a consequence of (6), the probability density of maximum output can be calculated as follows.

$$\hat{g}(z) = \begin{cases} \frac{ne^{n(z-\beta_1)}}{2^n} & \text{if} \quad z \leq \beta_1 \\ \frac{ne^{n(z-\beta_1)}(2-e^{-(z-\beta_1)})^{n-1}}{2^n} & \text{if} \quad \beta_2 \leq z \leq \beta_1 \end{cases}$$

Then, the expected value of maximum output becomes as follows.

$$\begin{aligned} & E(\max\{\beta_1+\epsilon_1,\ldots,\beta_n+\epsilon_n\}) \\ = \; & \frac{n}{2^n}\left(\int_{-\infty}^{\beta_1} e^{n(z-\beta_1)}z\,dz + \int_{\beta_1}^{\infty}(2-e^{-(z-\beta_1)})^{n-1}e^{-(z-\beta_1)}z\,dz\right) \\ = \; & \frac{1}{2^n}\left(\beta_1 - \frac{1}{n}\right) + \frac{n}{2}\sum_{k=0}^{n-1}\binom{n-1}{k}\left(-\frac{1}{2}\right)^k\left(\frac{1}{k+1}\right)\left(\beta_1 + \frac{1}{k+1}\right) \\ = \; & \beta_1 + \frac{1}{2}\sum_{k=0}^{n-1}\binom{n}{k+1}\left(-\frac{1}{2}\right)^k\left(\frac{1}{k+1}\right) - \left(\frac{1}{n2^n}\right) \end{aligned}$$

and the objective function in (7) changes into

$$\hat{\kappa}(\beta_1;n) = \beta_1 - \frac{n\beta_1^2}{2} + \frac{1}{2}\sum_{k=0}^{n-1}\binom{n}{k+1}\left(-\frac{1}{2}\right)^k\left(\frac{1}{k+1}\right) - \left(\frac{1}{n2^n}\right) - nu_0$$

As a result, $\hat{\kappa}'(\beta_1;n) = 1 - n\beta_1$ and, consequently, the optimal powers $\beta_i^* = e_i^* = 1/n$ are easily found. Then, each agent would be paid $u_0 + 1/(2n^2)$ and in the case that agent i yielded the best output, incentive schemes would be as follows.

$$W_i = \alpha_i + \gamma_i \max\{x_1,\ldots,x_i - 1, x_i + 1,\ldots,x_n\}$$
$$\alpha_j + \gamma_j \max\{x_1,\ldots,x_n\} = \alpha_j + \gamma_j x_i \quad \text{for} \quad j \neq i$$

But, there comes a problem to see the optimal value of

$$\hat{\kappa}(\beta_1^*;n) = \frac{1}{2}\sum_{k=0}^{n-1}\binom{n}{k+1}\left(-\frac{1}{2}\right)^k\left(\frac{1}{k+1}\right) - \left(\frac{2^{n-1}-1}{n2^n}\right) - nu_0$$

It is extremely difficult to analyze in terms of n because of involving combinations. In fact, a problem of arbitrariness might show up according to the value of u_0. [9] And to a certain degree, hiring more agents will work as a feasible solution to achieve the minimum requirement level unless $\hat{\kappa}(\beta_i^*;n)$ turns negative.

[9] A simple computer program in ANSI-C has found that expected value of maximum output grows steadily but the rate of growth diminishing as the number of agents increases until $n = 31$. overflow has happened for larger n's and so does $\hat{\kappa}(\beta_i^*;n)$ with $u_0 = 0$.

4 Discussions

4.1 Efficiency, Risk and Robustness

As mentioned, when the principal and agents are risk-neutral and there is only moral hazard with hidden action, first-best optima can be always reached. Though not a formal proof, a brief explanation goes as follows. Assume that principal's problem without informational asymmetry, i.e., the principal has perfect information. Then, it would assign an effort level optimal for itself to each agent and achieve first-best optima. Here is an example based on the 2-agent best-critical case.

$$\begin{aligned} \max_{e_1,e_2} \quad & E(\max\{x_1,x_2\}) - E(W_1+W_2) \\ \text{s. t.} \quad & E(W_i) - \frac{e_i^2}{2} \geq u_0 \\ & E(\max\{x_1,x_2\}) \geq k \end{aligned}$$

where for $i = 1, 2$

$$W_i = \alpha_i + \beta_i x_i + \gamma_i x_j$$

$$x_i = e_i + \epsilon_i + \eta$$

$$U_i = W_i - e_i^2/2$$

With individual rationality constraints binding, this problem is solved almost in the same way as that of previous cases. Though it is verified that wages and the principal's expected profit are the same without difficulty, β_i cannot be found. [10]

Changes in risk-sharing plan would affect the model and results drastically. If agents were risk-averse, the role of relative performance evaluation coefficient γ, and common shock η would be much more remarkable than it is under risk-neutrality. [11] But, it is certain that the first-best optimum cannot be obtained because of the tradeoff between optimal risk sharing and incentives. Linearity in incentive schemes is desirable in terms of robustness as well as implementability. Sophisticated schemes tend to be vulnerable to changes in assumptions and stochastic components.

4.2 Implications for R&D Policy

Several peculiar implications for regulation of R&D are claimed here. First, minimum requirement terms are identified as a potential source of asymmetric incentive schemes. In Levitt (1995), the principal is motivated to offer identical agents an asymmetric incentive scheme when the sampling effect, i.e., the variance of the idiosyncratic shocks, is relatively small to the effort levels. To specify the limiting cases, if the shock were zero a symmetric wage scheme would never be optimal because a lower powered agent could not produce the highest level of output. By the same token, an asymmetric scheme can never be optimal when idiosyncratic shocks converge to infinity, because agents will have the identical chance for the best output. In short, it is for the sake of more profit that the principal chooses an asymmetric scheme.

[10] If agents were risk-averse, they would want to be paid wholly by fixed α_i.
[11] An insurance effect. See Levitt (1995).

An asymmetric incentive powers, however, may be inevitable under such circumstances as supposed in this article. In fact, results that are analogous to Levitt is derived in the uniform distribution case, where choice of incentive schemes is dependent solely upon the consideration of profit. But if an idiosyncratic shock ϵ_i is from double exponential distribution, the principal will not be able to adopt a symmetric scheme that is absolutely more profitable than asymmetric schemes because of the high minimum requirement level. Moreover, excessively high minimum requirement level causes inappropriate suspension of a project which would be profitable. The neighborhood of the upper limit of the final output is very critical, then. And, at last social welfare will be increased.

In reality, minimum requirement terms such as due date are common. The implication of this article is that too severe terms may lower the principal's expected profit or even suspend a promising project. Although the minimum requirement is implicitly assumed to be exogenous, it can be regarded as not alterable once set by the principal before setting out the project. Thus, when the principal itself is responsible for arranging the minimum requirement, it should be serious not to choose an undesirable one.

For an realistic example, suppose a R&D project at which an immediate due date is designated as a minimum requirement. If the principal thinks that it is impossible for any agent to accomplish it within given time period(ex ante), it will be held back. Otherwise, the length of time that remain by the due date determines whether symmetric powers or asymmetric ones will be chosen. Once the incentive schemes are offered to agents, they decide how hard they are going to work. The more the power, the harder they will work. Paid by its own output, each agent will not shirk; so, optimal levels of effort are assured though their wages are not related to the final output. After the first output comes out from an agent, the project is terminated and agents are paid. [12]

5 Conclusions

This study has examined the influence which a minimum requirement term could have on principal-agent problems. It is revealed that a minimum requirement term to be satisfied could make significant changes in optimal incentive schemes as well as principal's profit under some conditions. It may also reduce social welfare by restricting incentive powers to less attractive ones for the principal. Discussions in detail have already been were given.

Compared to cases in which the final output is the sum of outputs, best-critical cases are affected more remarkably by the existence of the term. It is natural because in a best-critical case, an additional agent means an uncertain marginal increase in the final output but a certain expenditure for wages. In other words, the principal may hesitate about hiring one more agent because it cannot assure the principal of increase in the final output. On the other hand, in a sum-critical case, a minimum requirement level could be met easily by more agents. For instance, the principal may hire enough agents to satisfy minimum

[12] As a matter of fact the problem of rewarding agents does arise. For the discussion about the way that the principal should pay agents whose outputs were not the first, see Olsen (1993), pp.531-532.

requirements and organize agents to take advantage of efficiency from division of labor.

Thus, a core implication in this article is that when only the best output counts, a minimum requirement can be identified as a determinant of incentive schemes, and an excessively high minimum requirement on R&D activities can cast negative effect on social welfare.

There are several strong assumptions and cases to be covered in the article. Further studies are expected along following aspects. Firstly, the assumption on identical agents is too strong and restrictive. Applying adverse selection may alter the incentive scheme for agents to report their type truthfully and the whole results subsequently. Secondly, other risk-sharing plans are to be considered thereafter. Introduction of risk-averse agents means that the risk-neutral principal has to bear all the risk. Other basic changes, however, are also anticipated. (See Levitt(1995)) Finally, cases for only two probability distributions are provided here. It was because of the problem on its tractability that normal distribution was excluded initially. But in terms of its empirical usefulness and frequent occurrence, consideration of normal distribution would certainly contribute toward the study of minimum requirement in the aspect of practical implication.

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