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Application of the Operator-Algebra Method to the Quantum Limit Cyclotron Resonance for the Electron-Phonon System

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Abstract

The cyclotron resonance lineshape function in the electron-phonon systems is obtained by using the operator-algebra method. By keeping the higher order terms of the interaction in the energy denominators, the result could be applied to strong scattering case. We also could avoid the divergent terms in the perturbative expansions by utilizing different approach with some other authors

1. Introduction

Semiconductors in a high magnetic field display many interesting and complex spectral patterns like the zeeman spectra of large atoms or molecules. The belt of energies which form the conduction band of a semiconductor split into a large number of thin strips(Landau subbands)under magnetictic fields. The interpretation of the measured magneto-transport spetra rests on the microscopic calculations of the centroid and the width of these split bands. In the present article we consider the usual model of a semiconductor described by parabolic energy bands in which a system of electrons all possessing the same spherical effective mass m are moving. The material under consideration is in a constant high magnetic field, denoted by B,

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applied in the z-direction of the cartesian coordinate system.

The resonant response of the electrons with a laser field of frequency ω, called the cyclotron resonance, enables us to meaure directly the effective masses of electrons and holes. Before long, however, people got interested also in cyclotron resonance line-width. The cyclotron resonance lineshape shoud be infinitely sharp in the absence of interactions and the linebroadening, therefore, is generated by the scattering processes. The linewidth stands for the inverse of the carrier transport relaxation time and, roughly speaking, indicates the total sum of probabilities of various scatterings. This means that study of cyclotron resonance is quite useful not only for studying electronic structures in solids(1,2), but also for studying carrier scattering mechanisms (3-14).

A number of experimental studies of cyclotron resonance lineshape function (CRLSF) have been done on various semiconductors, and the experimental results have been analized utilizing Green's function methods (15-17), diagram methods(18-22), or projection operator methods(23-28). Among these theories, Kubo formalism(29) combined with the projection operator method of Kawabata(23), Choi et al(26,28) and Argyres et al(24,25) appear to be comparatively rigorous and formal.

Kawabata adopted Mori's projection operator method(30-34) to evaluate the current-current correlation function and derived the formula for the cyclotron resonance absorption lineshape on the basis of the generalized Langevin equation of Brownian motion, starting from the Kubo formula for electrical conductivity. His theory, however, is limited to the incoherent elastic scatterings and therefore is not applicable for the strong interaction case.

Fujita and Lodder(18,19) studied CRLSF by applying the proper connected diagram enpansion method to the Kubo formula. Latter, Prasad(35) extend the formula to the screened coulomb potential scattering, using the same method.

Argyres and Sigel(25) developed a theory with the use of a projection operator technique and they claimed that the pertubative expansions used in sometheories(18,19,23) are not valid at the peaks of the absorption lines. (This shall be discussed more later).

In 1986, Suzuki(22) obtained the formulas of cyclotron resonance power absorption spectral lines for an electron-phonon system in the case of weak incoherent and strong coherent scattering limits by using the resolvent superoperator method.

In 1990, Choi et al(28) developed a theory of CRLSF for an electron-phonon system with the help of the projection operator method and applied it to obtaining the cyclotron resonance linewidth for the piezoelectric

polaron system in the case of adiabatic and non adiabatic scattering processes. Their results agree well with the experimental results.

Recently, Kobori et al(13) has studied CRLSF in the quantum limit for impurity and carrier-carrier scatterings in Ge, InSb and GaAs and for phonon scatterings in Ge,Si, CdS and InSb. They obtained cyclotron resonance linewidth in the quantum limit as a function of temperature and magnetic field.

It is to be noted, however, that all these theoretical investigations have produced a bewildering variety of results. So the situation in quantum limit condition still remains unclear. The origin of discrepancy among these theories may be traced back to the ways the perturbative expansion is performed. As mentioned earlier in this section in connection with these perturbative expansion, Argyres and Sigel claimed that the correct CRLSF expression can not be expanded in powers of λ, because some terms of order λ^n with $n \geq 3$ diverge for frequences near the cyclotron frequency i.e., the expansion of CRLSF breaks down for $\omega = \omega_c$.

Argyres and sigel introduced an operator Δ to isolate the divergent terms.

Thus we think the proper way of looking for the explicit formula for the CRLSF rests in the calculation of the resolvent operator $(\omega - L)^{-1}$, L being the Liouville operator for the system. Here we solve the resolvent operator in a continued fraction form using projection operator to obtain the CRLSF for the phonon scattering case and discuss the removal of the divergence.

2. Conductivity

We present in this section an expression for the conductivity tensor of dynamically independent electrons in a uniform magnetic field. We assume that the applied microwave of amplitude E and frequency ω is circularly polarized in the x-y plane such that

$$E_x(t) = E\cos\omega t,\ E_y(t) = E\sin\omega t,\ E_z = 0. \tag{2.1}$$

Then the average power absorbed by the system is given by[18]

$$P(\omega) = \frac{1}{2} E^2\ Re\ \sigma_{+-}(\omega) \tag{2.2}$$

where Re $\sigma_{+-}(\omega)$ is the real part of the complex conductivity tensor expressed in kubo's current correlation integral formula as[18]

$$\sigma_{+-}(\omega) = \frac{1}{\Omega}\int_0^\infty e^{-i\omega}\,dt \int_0^\beta d\beta_1\ T_r\,[\rho J^-(-i\beta_1) J^+(t)] \tag{2.3}$$

$$J^\pm = J_x \pm i J_y \tag{2.4}$$

Here Ω is the volume of the system, $\beta \equiv (K_B T)^{-1}$, J_x and J_y, respectively, are the x and y components

of the total current operator in the many body formalism, $J(t)$ is the time dependent total current operator in the Heisenberg picture, ρ is the grand canonical density operator normalized to unity, the symbol Tr means the grand ensemble many body trace, and we use units in which $\hbar=1$

We consider an electron-phonon system in the presence of a constant magnetic field $\vec{B}$ applied along the z-axis which is perpendicular to $\vec{E}(t)$ and characterized by the vector potential $\vec{A}$ such that $\vec{B}=\vec{\nabla}\times\vec{A}$. It is convenient to use the Landau gauge for the vetor potential $\vec{A}=(0,Bx,0)$, the Hamiltonian H of a system of non-interacting electrons in a phonon field is given by

$$H=\sum_i h^{(i)}+H_p \tag{2.5}$$

$$h=H_0+V \tag{2.6}$$

$$H_0=\frac{(\vec{p}+e\vec{A})^2}{2m} \tag{2.7}$$

$$V=\sum_q(\gamma_{\vec{q}}\, b_{\vec{q}}+\gamma_{\vec{q}}^{\dagger} b_{\vec{q}}^{\dagger}) \tag{2.8}$$

$$\gamma_{\vec{q}}=C_{\vec{q}}\exp(i\vec{q}\cdot\vec{r}) \tag{2.9}$$

$$H_p=\sum_q \omega_{\vec{q}}\, b_{\vec{q}}^{\dagger} b_{\vec{q}} \tag{2.10}$$

Where $b_{\vec{q}}^{\dagger}$ and $b_{\vec{q}}$, respectively, are the creation and annihilation operators of a phonon with momentum $\vec{q}$ and energy $\omega_{\vec{q}}$, $C_{\vec{q}}$ is the interaction operator and $\vec{p}$ is the momentum of a conduction electron with effective mass m. Then the unperturbed electron Hamiltonian H_o takes the form

$$H_0=\left[P_x^2+(P_y+m\omega_c X)^2+P_z^2\right]/2m \tag{2.11}$$

Where $\omega_c\equiv eB/m$ is the cyclotron frequency. The energy eigenvalues E_α and eigenfunctions of H_0 are specified by the oscillator quantum numbers $N(N=0,1,2,...)$ and the wave vectors $\vec{k}=(k_x,k_y)$ and, respectively, given by

$$E_\alpha=E_{N_\alpha k_\alpha}\equiv(N+\frac{1}{2})\,\omega_c+\frac{k_z^2}{2m} \tag{2.12}$$

$$|\alpha>=\Psi_{N_\alpha,k_\alpha}(\vec{r}) \tag{2.13}$$

$$\Psi_{N_\alpha,k_\alpha}(\vec{r})=(L_y L_z)^{-1/2}\exp(iyk_y+izk_z)\Phi_N(x-X) \tag{2.14}$$

$$\Phi_N(x)=(2^N N!r_0\sqrt{\pi})^{-1/2}H_N(x/r_0)\exp\{-x^2/(2r_0^2)\} \tag{2.15}$$

where $r_0\equiv(m\omega_c)^{-1/2}$, $X\equiv -k_y/m\omega_c$, H_N is the N-th Hermite polynomial, and L_y and L_z, respectively, are y and z directional normalization lengths. In the following we shall adopt the notation $|\alpha+1>$ to denote the state $|N_\alpha+1,\vec{k}_\alpha>$ if $|\alpha>=|N_\alpha,\vec{k}_\alpha>$. We ignore the spin of the electrons. We see from Eq.(2.12) that the motion of the electron is quantized in the x-y plane and quasicontinuous in the z-

direction. We, then, can rewrite $\sigma_{+-}(\omega)$ in the single electron expression :

$$\sigma_{+-}(\omega)=\frac{1-\exp^{-\beta\omega}}{\omega_c \Omega}\sum_{\alpha} f(E_\alpha)\ \{1-f(E_\alpha+\omega_c)\ \}<\alpha\mid j^-\mid \alpha+1><\tilde{F}_\alpha(\omega)>_{ph} \tag{2.16}$$

Here $f(E)$ stands for the Fermi distribution function and $\tilde{F}_\alpha(\omega)$ is the Fourier-Laplace transform(FLT) of $<\alpha+1\mid j^+(t)\mid \alpha>$ defined by

$$\tilde{F}_\alpha(\omega)\equiv\int_0^\infty dt\ e^{\varepsilon t}e^{-i\omega}<\alpha+1\mid j^+(t)\mid \alpha> \tag{2.17}$$

which yields the lineshape function, as will be clarified later. The time dependent single electron current operator is defined by

$$j^+(t)=e^{iLt}\ j^+ \tag{2.18}$$

$$L=L_0+L_1+L_p \tag{2.19}$$

where L_0, L_1 and L_p being the Liouville operators corresponding to the unperturbed single electron Hamiltonian H_0, the scattering potential V and the phonon hamiltonian H_p. The angular bracket $<\ >_{ph}$ denotes the average over the phonon background, but hereafter we omit this average notation for convenience. It should be noted that $<\alpha\mid j^+\mid \beta>=(<\beta\mid j^-\mid \alpha>)^*=j^+_\beta\ \delta_{\alpha,\beta+1}$ holds for arbitrary states.

3. Calculation of time correlation function

The time evolution of a dynamical variable j^+ in the Heisenberg picture is formally given by Eq.(2.18) For our purpose we construct a biorthogonal set of vectors and the corresponding projection operators which differs from Mori's operator but the theories are self consistent[31]. The quantity j^+, which generates the successive basis set, forms a Hilbert space. The projection of a current operator $j^+(t)$ onto the j^+ axis is given by

$$P_0\, j^+(t)\equiv\frac{(A_\alpha, j^+(t))}{(A_\alpha, j^+)}\, j^+ \tag{3.1}$$

Where the inner product of two variables in the fixed electron state is defined as [23,35]

$$\begin{aligned}(A_\alpha,\ j^+(t))&\equiv tr\ \{A_\alpha, j^+(t)\ \}\\ &=\sum_\beta <\beta\mid a^+_\alpha a_{\alpha+1}\, j^+(t)\mid \beta>=<\alpha+1\mid j^+(t)\mid \alpha>\end{aligned} \tag{3.2}$$

Where $A_\alpha=a^+_\alpha a_{\alpha+1}$, $a^+_\alpha\,(a_\alpha)$ being the creation(annihilation) operator for the electron state $\mid \alpha>$.

We also define the normalized time correlation function as

$$\Xi_0(t) \equiv (A_\alpha, j^+(t)) / (A_\alpha, j^+) \tag{3.3}$$

By taking FLT of the above equation, we have

$$\tilde{\Xi}_0(\omega) = FLT\left[(A_\alpha, j^+(t)) / (A_\alpha, j^+)\right]$$

$$= (A_\alpha, \frac{1}{i(\omega - L)} j^+) / (A_\alpha, j^+) \tag{3.4}$$

which can be rewritten into the following form :

$$\tilde{\Xi}_0(\omega) = (A_\alpha \frac{-i}{\omega - LP_0 - LQ_0} j^+) / (A_\alpha, j^+)$$

$$= (A_\alpha \{\frac{-i}{\omega - LQ_0} + \frac{1}{\omega - LQ_0} LP_0 \frac{-i}{\omega - L} \} j^+) / (A_\alpha, j^+) \tag{3.5}$$

where $Q_0 = 1 - P_0$, $\omega - L = \omega - LP_0 - LQ_0$ and the operator identity[36, 37]

$$\frac{1}{A+B} = \frac{1}{A} - \frac{1}{A} B \frac{1}{A+B} \tag{3.6}$$

have been used. The first part of Eq. (3.5) can be calculated as

$$(A_\alpha, \frac{-i}{\omega - LQ_0} j^+) = (A_\alpha, -i \{\frac{1}{\omega} + \frac{1}{\omega} LQ_0 \frac{1}{\omega} + \cdots\cdots \} j^+)$$

$$= -i (A_\alpha, j^+) / \omega \tag{3.7}$$

Since $(A_\alpha, LQ_0 j^+) = 0$. The second part of Eq. (3.5) can be calculated by using

$$P_0 (\frac{-i}{\omega - L} j^+) = \frac{(A_\alpha, \frac{-i}{\omega - L} j^+)}{(A_\alpha, j^+)} j^+ = \tilde{\Xi}_0(\omega) j^+ \tag{3.8}$$

Inserting Eqs. (3.7) and(3.8) into Eq. (3.5), we obtain

$$\tilde{\Xi}_0(\omega) = \left[i\omega - \frac{(A_\alpha, iLj^+)}{(A_\alpha, j^+)} - \frac{(A_\alpha, iLQ_0(\omega - LQ_0)^{-1} Lj^+)}{(A_\alpha, j^+)} \right]^{-1} \tag{3.9}$$

In order to simplify the problem we assume that L is Hermitian. We also consider the relations $Q_0(\omega - LQ_0)^{-1} = \sum_{n=0}^{\infty} \frac{1}{\omega} (Q_0 L \frac{1}{\omega})^n Q_0 = (\omega - Q_0 L)^{-1} Q_0$ and $(P_0 LA_\alpha, (\omega - Q_0 L)^{-1} Q_0 Lj^+) = 0$. We then obtain

$$\tilde{\Xi}_0(\omega)=\left[i\omega-\frac{(A_\alpha, iLj^+)}{(A_\alpha, j^+)}+\frac{(iQ_0LA_\alpha, \frac{1}{i(\omega-Q_0L)}iQ_0Lj^+)}{(A_\alpha, j^+)}\right]^{-1}$$

$$=[i\omega-i\omega_0+\tilde{\Xi}_1(\omega)\Delta_1]^{-1} \tag{3.10}$$

where

$$i\omega_0\equiv(A_\alpha, iLj^+)/(A_\alpha, j^+) \tag{3.11}$$

$$\tilde{\Xi}_1(\omega)\equiv(f_1, \frac{1}{i(\omega-\hat{L}_1)}g_1)/(f_1, g_1) \tag{3.12}$$

$$f_1\equiv i\hat{L}_1A_\alpha \tag{3.13}$$

$$g_1\equiv i\hat{L}_1j^+ \tag{3.14}$$

$$\hat{L}_1\equiv Q_0L \tag{3.15}$$

$$\Delta_1\equiv(f_1, g_1)/(A_\alpha, j^+) \tag{3.16}$$

$\tilde{\Xi}_1(\omega)$ is similar to $\tilde{\Xi}_0(\omega)$ in form and thus the demominator $(\omega-\hat{L}_1)^{-1}$ in Eq.(3.12) can be calculated further with the same procedure as Eqs.(3.4)~(3.10). The result is

$$\tilde{\Xi}_1(\omega)=\left[i\omega-i\omega_1\tilde{\Xi}_2(\omega)\Delta_2\right]^{-1} \tag{3.17}$$

where

$$i\omega_1\equiv(f_1, i\hat{L}_1g_1)/(f_1, g_1)$$

$$\tilde{\Xi}_2(\omega)\equiv(f_2, \frac{1}{i(\omega-\hat{L}_2)}g_2)/(f_2, g_2) \tag{3.19}$$

$$f_2\equiv i\hat{L}_2f_1 \tag{3.20}$$

$$g_2\equiv i\hat{L}_2g_1 \tag{3.21}$$

$$\hat{L}_2\equiv Q_1\hat{L}_1 \tag{3.22}$$

$$Q_1\equiv(1-P_1) \tag{3.23}$$

$$P_1Y\equiv(f_1, Y)\cdot(f_1, g_1)^{-1}\cdot g_1 \tag{3.24}$$

$$\Delta_2\equiv(f_1, g_2)\cdot(f_1, g_1)^{-1} \tag{3.25}$$

Y being an arbitrary operator. Thus generally we can rewrite as $\tilde{\Xi}_0(\omega)$ as follows :

$$\tilde{\Xi}_0(\omega)=\cfrac{1}{i\omega-i\omega_0+\cfrac{\Delta_1}{i\omega-i\omega_1+\cfrac{\Delta_2}{i\omega-i\omega_2+\cdots\cdots}}} \tag{3.26}$$

Now Eq. (2.17) can be rewritten, by using Eqs. (3.2), (3.3)and (3.26), as

$$\tilde{F}_\alpha(\omega)=\int_0^\infty dt\exp^{-i\omega t}<\alpha+1\mid j^+(t)\mid\alpha>$$

$$=\frac{j^+_\alpha}{i\omega-i\omega_0+\tilde{\Gamma}_\alpha(\omega)} \tag{3.27}$$

where

$$j^+_\alpha\equiv<\alpha+1\mid j^+\mid\alpha> \tag{3.28}$$

$$\tilde{\Gamma}_\alpha(\omega)\equiv\frac{\Delta_1}{i\omega-i\omega_1} \tag{3.29}$$

The iteration could by continued further in Eq. (3.29), but we stop here in approximation. From Eqs. (2.16) and (3.27) we have

$$\sigma_{+-}(\omega)=\frac{1-\exp^{-\beta\omega_c}}{\omega_c\Omega}\sum_\alpha f(E_\alpha)\{1-f(E_\alpha+\omega_c)\}\frac{\mid j^+_\alpha\mid^2}{i\omega-i\omega_0+\tilde{\Gamma}_\alpha(\omega)} \tag{3.30}$$

where the ω_0 in the $\mid\alpha\rangle$ representation in Eq. (3.11) is given by

$$i\omega_0=i\omega_c+V_{\alpha+1,\alpha+1}-V_{\alpha,\alpha} \tag{3.31}$$

which yields $\omega_0=\omega_c$ if we assume $<V_{\alpha\alpha}>_{ph}=0$. In the above equation we used the notation $X_{\alpha\beta}\equiv<\alpha\mid X\mid\beta>$.

4. Lineshape Function

Let us consider the collision term $\tilde{\Gamma}_\alpha(\omega)$, called CRLSF, which essentially determines the resonance lineshape. In order to calculate the CRLSF, we should start with the calculation of the factor (f_1, g_1) which appears in the numerator of Δ_1 and the denominator of ω_1. Here we will make use of the following relations :

$$Q_0L_0j^+=0 \tag{4.1}$$

$$(L_0A_\alpha, Q_0Yj^+)=0 \tag{4.2}$$

$$L_0j^+=\omega_c j^+ \tag{4.3}$$

Then from Eqs. (3.13) and (3.14) we immediately have

$$(f_1, g_1)=(VA_\alpha, Vj^+)-(VA_\alpha, P_0Vj^+)-(VA_\alpha, j^+V)+(VA_\alpha, P_0j^+V)$$
$$-(A_\alpha V, j^+)+(A_\alpha V, P_0Vj^+)+(A_\alpha V, j^+V)-(A_\alpha V, P_0j^+V) \tag{4.4}$$

By taking into account Eqs. (3.1) and (3.2) we have

$$\Delta_1=(f_1, g_1)/(A_\alpha, j^+)$$

$$= (j_\alpha^+)^{-1}\Big[\sum_{\beta \gtrless \alpha} j_\beta^+ V_{\beta,\alpha} V_{\alpha+1,\beta+1} - \sum_{\beta \gtrless \alpha} j_\alpha^+ V_{\beta,\alpha} V_{\alpha,\beta}$$

$$- \sum_{\beta \gtrless \alpha+1} j_\alpha^+ V_{\alpha+1,\beta} V_{\beta,\alpha+1} + \sum_{\beta \gtrless \alpha+1} j_{\beta-1}^+ V_{\alpha+1,\beta} V_{\beta-1,\alpha}\Big] \qquad (4.5)$$

Averaging over the phonon distribution can be carried out by considering $\langle b_{\vec{q}}^{\ddagger} b_{\vec{q}} \rangle_{ph} = n_{\vec{q}}\ \delta_{\vec{q}\,\vec{q}}$, and $\langle b_{\vec{q}} b_{\vec{q}}^{\ddagger} \rangle_{ph} = (1+n_{\vec{q}})\ \delta_{\vec{q}\,\vec{q}}$, $n_{\vec{q}}$ being the plank distribution function. As a result we have

$$\Delta_1 = (j_\alpha^+)^{-1}\Big[\sum_{\beta \gtrless \alpha} \hat{V}_1 + \sum_{\beta \gtrless \alpha+1} \hat{V}_2 + \sum_{\beta \gtrless \alpha} \hat{V}_3 + \sum_{\beta \gtrless \alpha+1} \hat{V}_4\Big] \qquad (4.6)$$

where

$$\hat{V}_1 = \sum_q (1+n_{\vec{q}})\Big[j_\beta^+ (\gamma_{\vec{q}})_{\beta,\alpha} (\gamma_{\vec{q}}^{\ddagger})_{\alpha+1,\beta+1} - j_\alpha^+ (\gamma_{\vec{q}})_{\beta,\alpha} (\gamma_{\vec{q}}^{\ddagger})_{\alpha,\beta}\Big] \qquad (4.7)$$

$$\hat{V}_2 = \sum_q (1+n_{\vec{q}})\Big[j_{\beta-1}^+ (\gamma_{\vec{q}})_{\alpha+1,\beta} (\gamma_{\vec{q}}^{\ddagger})_{\beta-1,\alpha} - j_\alpha^+ (\gamma_{\vec{q}})_{\alpha+1,\beta} (\gamma_{\vec{q}}^{\ddagger})_{\beta,\alpha+1}\Big] \qquad (4.8)$$

$$\hat{V}_3 = \sum_q n_{\vec{q}}\Big[j_\beta^+ (\gamma_{\vec{q}}^{\ddagger})_{\beta,\alpha} (\gamma_{\vec{q}})_{\alpha+1,\beta+1} - j_\alpha^+ (\gamma_{\vec{q}}^{\ddagger})_{\beta,\alpha} (\gamma_{\vec{q}})_{\alpha,\beta}\Big] \qquad (4.9)$$

$$\hat{V}_4 = \sum_q n_{\vec{q}}\Big[j_{\beta-1}^+ (\gamma_{\vec{q}}^{\ddagger})_{\alpha+1,\beta} (\gamma_{\vec{q}})_{\beta-1,\alpha} - j_\alpha^+ (\gamma_{\vec{q}}^{\ddagger})_{\alpha+1,\beta} (\gamma_{\vec{q}})_{\beta,\alpha+1}\Big] \qquad (4.10)$$

On the other hand, the numerator of $i\omega_1$ (Eq. (3.18)) can be expanded by taking into account $Q_0 Q_0 = Q_0$ and Eqs. (4.1) and (4.3), and considering only the second order in V.

the numerator of $i\omega_1 = (f_1, iQ_0 L g_1)$

$$= i\Big[(VA_\alpha, H_0 Vj^+) - 2(VA_\alpha, H_0 j^+ V) - 2(AV_\alpha, Vj^+ H_0) + (VA_\alpha, j^+ VH_0)$$

$$-(A_\alpha V, H_0 Vj^+) + 2(A_\alpha V, H_0 j^+ V) + 2(A_\alpha V, Vj^+ H_0) - (A_\alpha V, j^+ VH_0)$$

$$-(VA_\alpha, P_0 H_0 Vj^+) + 2(VA_\alpha, P_0 H_0 j^+ V) + 2(VA_\alpha, P_0 Vj^+ H_0) - (VA_\alpha, P_0 j^+ VH_0)$$

$$+(A_\alpha V, P_0 H_0 Vj^+) - 2(A_\alpha V, P_0 H_0 j^+ V) - 2(A_\alpha V, P_0 Vj^+ H_0) + (A_\alpha V, P_0 j^+ VH_0)$$

$$+(VA_\alpha, H_p VJ^+) - (VA_\alpha, H_p j^+ V) - (VA_\alpha, Vj^+ H_p) + (VA_\alpha, j^+ VH_p)$$

$$-(A_\alpha V, H_p VJ^+) + (A_\alpha V, H_p j^+ V) + (A_\alpha V, Vj^+ H_p) - (A_\alpha V, j^+ VH_p)$$

$$-(VA_\alpha, P_0 H_p Vj^+) + (VA_\alpha, P_0 H_p j^+ V) + (VA_\alpha, P_0 Vj^+ H_p) - (VA_\alpha, P_0 j^+ VH_p)$$

$$+(A_\alpha V, P_0 H_p Vj^+) - (A_\alpha V, P_0 H_p j^+ V) - (A_\alpha V, P_0 Vj^+ H_p) + (A_\alpha V, P_0 j^+ VH_p)$$

$$+(VA_\alpha, VH_0 j^+)+(VA_\alpha, j^+ H_0 V)-(A_\alpha V, VH_0 j^+)-(A_\alpha V, j^+ H_0 V)$$

$$-(VA_\alpha, P_0 VH_0 j^+)-(VA_\alpha, P_0 j^+ H_0 V)+(A_\alpha V, P_0 VH_0 j^+)+(A_\alpha V, P_0 j^+ H_0 V)\Big]$$

$$+i\omega_c\Big[(VA_\alpha, Vj^+)-(VA_\alpha, j^+ V)-(A_\alpha V, Vj^+)+(A_\alpha V, j^+ V)$$

$$-(VA_\alpha, P_0 Vj^+)+(VA_\alpha, P_0 j^+ V)+(A_\alpha V, P_0 Vj^+)-(A_\alpha V, P_0 j^+ V)\Big] \qquad (4.11)$$

For the sake of algebraic convenience, we divide these terms into four parts in the following way :

$$Part(I)=(1st+9th)+(3rd+11th)+(17th+25th)+(19th+27th)+(33rd+37th)+(41st+45th)$$

$$=i\sum_{\beta\gtrless\alpha} j^+_\beta\Big[(2\omega+E_{\alpha+1}-E_\beta)V_{\beta,\alpha}V_{\alpha+1,\beta+1}+V_{\beta,\alpha}H_p V_{\alpha+1,\beta+1}-V_{\beta,\alpha}V_{\alpha+1,\beta+1}H_p\Big] \qquad (4.12)$$

$$Part(\text{Ⅱ})=(2nd+10th)+(4th+12nd)+(18th+26th)+(20th+28th)+(34th+38th)+(42nd+46th)$$

$$=-i\sum_{\beta\gtrless\alpha} j^+_\alpha\Big[(2\omega+E_{\alpha+1}-E_\beta)V_{\beta,\alpha}V_{\alpha,\beta}+V_{\beta,\alpha}H_p V_{\alpha,\beta}-V_{\beta,\alpha}V_{\alpha,\beta}H_p\Big] \qquad (4.13)$$

$$Part(\text{Ⅲ})=(5th+13rd)+(7th+15th)+(23rd+31st)+(29th+21st)+(35th+39th)+(43rd+47th)$$

$$=-i\sum_{\beta\gtrless\alpha+1} j^+_\alpha\Big[(2\omega+E_\beta-E_\alpha)V_{\alpha+1,\beta}V_{\beta,\alpha+1}+V_{\alpha+1,\beta}H_p V_{\beta,\alpha+1}-V_{\alpha+1,\beta}V_{\beta,\alpha+1}H_p\Big] \qquad (4.14)$$

$$Part(\text{Ⅳ})=(6th+14th)+(18th+16th)+(22nd+30th)+(24th+32rd)+(36th+40th)+(44th+48th)$$

$$=i\sum_{\beta\gtrless\alpha+1} j^+_{\beta-1}\Big[(2\omega+E_\beta-E_\alpha)V_{\alpha+1,\beta}V_{\beta-1,\alpha}+V_{\alpha+1,\beta}H_p V_{\beta-1,\alpha}-V_{\alpha+1,\beta}V_{\beta-1,\alpha}H_p\Big] \qquad (4.15)$$

Since the denominator of $i\omega_1,(f_1,g_1)$ is identical with the numerator of Δ_1, we can adjust the terms of ω_1 as follows :

$$\omega_1=\left(\sum_{\beta\gtrless\alpha}\hat{V}_1 E_1\sum_{\beta\gtrless\alpha+1}\hat{V}_2 E_2+\sum_{\beta\gtrless\alpha}\hat{V}_3 E_3+\sum_{\beta\gtrless\alpha+1}\hat{V}_4 E_4\right)/j^+_\alpha\Delta_1 \qquad (4.16)$$

where

$$E_1 = 2\omega + E_{\alpha+1} - E_\beta + \omega_q^{\rightarrow} \tag{4.17}$$

$$E_2 = 2\omega + E_\beta - E_\alpha + \omega_q^{\rightarrow} \tag{4.18}$$

$$E_3 = 2\omega + E_{\alpha+1} - E_\beta - \omega_q^{\rightarrow} \tag{4.19}$$

$$E_4 = 2\omega + E_\beta - E_\alpha - \omega_q^{\rightarrow} \tag{4.20}$$

Therefore, by substituting Δ_1 and $i\omega_1$, in Eqs.(4.6) and (4.16) into Eq.(3.29)(CRLSF, $\tilde{\Gamma}_\alpha(\omega)$)we obtain

$$i\tilde{\Gamma}_\alpha(\omega) = \sum_q (1+n_q^{\rightarrow}) \left[\sum_{\beta \gtrless \alpha} \frac{(\gamma_q^{\rightarrow})_{\beta,\alpha} \left\{ (\gamma_q^{\ddagger})_{\alpha,\beta} - \frac{j_\beta^+}{j_\alpha^+} (\gamma_q^{\ddagger})_{\alpha+1,\beta+1} \right\}}{\omega + E_{\alpha+1} - E_\beta + \omega_q^{\rightarrow} + \bar{\Xi}_2} \right.$$

$$\left. + \sum_{\beta \gtrless \alpha+1} \frac{(\gamma_q^{\rightarrow})_{\alpha+1,\beta} \left\{ (\gamma_q^{\ddagger})_{\beta,\alpha+1} - \frac{j_{\beta-1}^+}{j_\alpha^+} (\gamma_q^{\ddagger})_{\beta-1,\alpha} \right\}}{\omega + E_\beta - E_\alpha + \omega_q^{\rightarrow} + \bar{\Xi}_2} \right]$$

$$+ \sum_q n_q^{\rightarrow} \left[\sum_{\beta \gtrless \alpha} \frac{(\gamma_q^{\ddagger})_{\beta,\alpha} \left\{ (\gamma_q^{\rightarrow})_{\alpha,\beta} - \frac{j_\beta^+}{j_\alpha^+} (\gamma_q^{\rightarrow})_{\alpha+1,\beta+1} \right\}}{\omega + E_{\alpha+1} - E_\beta - \omega_q^{\rightarrow} + \bar{\Xi}_3} \right.$$

$$\left. + \sum_{\beta \gtrless \alpha+1} \frac{(\gamma_q^{\ddagger})_{\alpha+1,\beta} \left\{ (\gamma_q^{\rightarrow})_{\beta,\alpha+1} - \frac{j_{\beta-1}^+}{j_\alpha^+} (\gamma_q^{\rightarrow})_{\beta-1,\alpha} \right\}}{\omega + E_\beta - E_\alpha - \omega_q^{\rightarrow} + \bar{\Xi}_4} \right] \tag{4.21}$$

where

$$\bar{\Xi}_1 = \left[\left(\sum_{\beta \gtrless \alpha+1} \hat{V}_2 E_2 - \sum_{\beta \gtrless \alpha+1} \hat{V}_2 \sum_{\beta \gtrless \alpha} E_1 \right) + \left(\sum_{\beta \gtrless \alpha} \hat{V}_3 E_3 - \sum_{\beta \gtrless \alpha} \hat{V}_3 \sum_{\beta \gtrless \alpha} E_1 \right) \right.$$

$$\left. + \left(\sum_{\beta \gtrless \alpha+1} \hat{V}_4 E_4 - \sum_{\beta \gtrless \alpha+1} \hat{V}_4 \sum_{\beta \gtrless \alpha} E_1 \right) - \theta_1 \right] / j_\alpha^+ \Delta_1 \tag{4.22}$$

$$\bar{\Xi}_2 = \left[\left(\sum_{\beta \gtrless \alpha+1} \hat{V}_2 E_1 - \sum_{\beta \gtrless \alpha+1} \hat{V}_2 \sum_{\beta \gtrless \alpha+1} E_2 \right) + \left(\sum_{\beta \gtrless \alpha} \hat{V}_3 E_3 - \sum_{\beta \gtrless \alpha} \hat{V}_3 \sum_{\beta \gtrless \alpha} E_2 \right) \right.$$

$$\left. + \left(\sum_{\beta \gtrless \alpha+1} \hat{V}_4 E_4 - \sum_{\beta \gtrless \alpha+1} \hat{V}_4 \sum_{\beta \gtrless \alpha} E_2 \right) - \theta_2 \right] / j_\alpha^+ \Delta_1 \tag{4.23}$$

$$\bar{\Xi}_3 = \left[\left(\sum_{\beta \gtrless \alpha} \hat{V}_1 E_1 - \sum_{\beta \gtrless \alpha} \hat{V}_1 \sum_{\beta \gtrless \alpha} E_3 \right) + \left(\sum_{\beta \gtrless \alpha+1} \hat{V}_2 E_2 - \sum_{\beta \gtrless \alpha+1} \hat{V}_2 \sum_{\beta \gtrless \alpha} E_3 \right) \right.$$

$$\left. + \left(\sum_{\beta \gtrless \alpha+1} \hat{V}_4 E_4 - \sum_{\beta \gtrless \alpha+1} \hat{V}_4 \sum_{\beta \gtrless \alpha} E_3 \right) - \theta_3 \right] / j_\alpha^+ \Delta_1 \tag{4.24}$$

$$\bar{\Xi}_4=\left[\left(\sum_{\beta\neq\alpha}\hat{V}_1E_1-\sum_{\beta\neq\alpha}\hat{V}_1\sum_{\beta\neq\alpha+1}E_4\right)+\left(\sum_{\beta\neq\alpha+1}\hat{V}_2E_2-\sum_{\beta\neq\alpha+1}\hat{V}_2\sum_{\beta\neq\alpha+1}E_4\right)\right.$$

$$\left.+\left(\sum_{\beta\neq\alpha}\hat{V}_3E_3-\sum_{\beta\neq\alpha}\hat{V}_3\sum_{\beta\neq\alpha+1}E_4\right)-\theta_4\right]/j_\alpha^+\Delta_1 \qquad (4.25)$$

and

$$\theta_1=\sum_q\sum_{\beta\neq\alpha}\sum_{(\alpha\neq\mu,\,\beta\neq\nu)}(1+n_q^{\rightarrow})(\gamma_q^{\rightarrow})_{\nu\mu}$$

$$\left\{j_\nu^+(\gamma_q^{\rightarrow})_{\mu+1,\,\nu+1}-j_\mu^+(\gamma_q^{\rightarrow})_{\mu\nu}\right\}\cdot(2\omega+E_{\alpha+1}-E_\beta+\omega_q^{\rightarrow}) \qquad (4.26)$$

$$\theta_2=\sum_q\sum_{\beta\neq\alpha+1}\sum_{(\alpha\neq\mu,\,\beta\neq\nu)}(1+n_q^{\rightarrow})(\gamma_q^{\rightarrow})_{\mu+1,\,\nu}$$

$$\left\{j_{\nu-1}^+(\gamma_q^{\rightarrow})_{\nu-1,\,\mu}-j_\mu^+(\gamma_q^{\rightarrow})_{\nu,\,\mu+1}\right\}\cdot(2\omega+E_\beta-E_\alpha+\omega_q^{\rightarrow}) \qquad (4.27)$$

$$\theta_3=\sum_q\sum_{\beta\neq\alpha}\sum_{(\alpha\neq\mu,\,\beta\neq\nu)}n_q^{\rightarrow}(\gamma_q^{\rightarrow})_{\nu\mu}$$

$$\left\{j_\nu^+(\gamma_q^{\rightarrow})_{\mu+1,\,\nu+1}-j_\mu^+(\gamma_q^{\rightarrow})_{\mu\nu}\right\}\cdot(2\omega+E_{\alpha+1}-E_\beta-\omega_q^{\rightarrow}) \qquad (4.28)$$

$$\theta_4=\sum_q\sum_{\beta\neq\alpha+1}\sum_{(\alpha\neq\mu,\,\beta\neq\nu)}n_q^{\rightarrow}(\gamma_q^{\rightarrow})_{\mu+1,\,\nu}$$

$$\left\{j_{\nu-1}^+(\gamma_q^{\rightarrow})_{\nu-1,\,\mu}-j_\mu^+(\gamma_q^{\rightarrow})_{\nu,\,\mu+1}\right\}\cdot(2\omega+E_\beta-E_\alpha-\omega_q^{\rightarrow}) \qquad (4.29)$$

The $(\beta\neq\nu,\ \alpha\neq\mu)$ in the summation means that the terms $(N_\alpha,\vec{k}_\alpha)=(N_\mu,\vec{k}_\mu)$ and/or $(N_\beta,\vec{k}_\beta)=(N_\nu,\vec{k}_\nu)$ should be excluded. Equation(4.21) is similar to the result of suzuki[22] in the sense that the higher order effects of the interactions are included in the energy denominators. But we can avoid the postulation of the nature done by suzuki in the expansion of the collision term[see Eq.(3.12) of (Ref. 22)]. This kind of appropriate approximation was also used in Fujita-Lodder's [18.19] paper obtained by the diagram approach [Eq.(5.16)in (Ref.18)].

5. Discussion and conclusion

Until now we have obtained the correlation function in the form of continued fraction, straight forwardly by expanding the resolvent operator $(\omega-L)^{-1}$ algebraically using the projection operator. And we applied this method in deriving the CRLSF for the electron-phonon systems.

Followings are the comparison with other theories. Kawabata calculated the integral equation of current-current correlation function with tracing the method of looking for Generalized Langevin equation of Mori. In the evaluation of the memory function, he replaced the Liouville operator L by L_0 [Eqs. (A.23) and (A.39) in (Ref.23)] and this led to the dynamics in the Born approximation which washed out the coherent scattering effects. This may not be correct in strongly correlated systems such as liquids, where a more accurate expansion of the collision operator is required.

Argyres and sigel[25] expanded the CRLSF $\widetilde{\Gamma}_\alpha(\omega)$ as follows [Eq. (2.31) in (Ref.25)] :

$$\widetilde{\Gamma}_\alpha(\omega) = -\frac{i}{j^+_\alpha}\left(\left[L_1 + L_1 G_0 P' L_1 + L_1 G_0 P' L_1 G_0 P' L_1 + \ldots\right] j^+\right)_\alpha \qquad (5.1)$$

where $G_0 = (\omega - L_0)^{-1}$ and $P' = 1 - P$. They obtained the CRLSF by approximation of cut off up to the second order term in Eq. (5.1). We see that this corresponds directly to Kawabata's approximation of $e^{iP'\cdot L} = e^{iL\cdot}$ in memory function calculation.

If we do expand the term $(\omega - Q_0 L)^{-1}$ in Eq. (3.10) as series type, the result will be reduced to Argyres and Sigel's expression. Further, they claimed that in the expansion of $\widetilde{F}_\alpha(\omega)$ in powers of λ, there are terms of order λ^n with $n \geq 3$ which break down for frequencies near the resonant cyclotron frequency ($\omega = \omega_c$) i.e., higher order terms always involve $G_0(\omega) j^+ = j^+ / \delta\omega$, ($\delta\omega = \omega - \omega_c$). Thus they isolated the divergent terms $\widetilde{\Gamma}_\alpha(\omega)$ in by introducing operator Δ. Fujita and Lodder[18] also discussed this collision process by using the diagram method. They eliminated the divergence near $\omega = \omega_c$ in the connected diagram expansion by introducing the collision factor g_z responsible for the line broadening. In other words, they restricted themselves only to the proper connected diagram expansion[Eqs. (3.14) and (3.17) in (Ref. 18)].

In the present work we could avoid the resonance divergent terms including $(\omega - L_0)^{-1}$ in the purturbative expansion, which can be observed with a close look at the structure of Eq. (3.26). This was performed by representing iteratively the resolvent operator $(\omega - L)^{-1}$ in the continued fraction form applying the projection operators P_0 and Q_0 on the Liouville operator L instead of separating L_0 and L_1 as shown in Eq. (3.5). That is, in accordance with how the exponential terms of the memory function are expanded in series or continued fraction type, the divergence terms $G_0(\omega) j^+ = j^+ / \delta\omega$ above the third order term appear or disappear.

On the other hand, Eq. (4.21) can be compared with the result of suzuki[22]. If we consider the exclusions factor in $\sum_\beta$ and $\theta = 1$, our result reduces to Suzuki's [Eq. (4.8) in (Ref.22)].

In this paper, we obtained the CRLSF for electron-phonon systems with a more precise calculation, which is applicable to the strong scattering case and could avoid the divergent terms without introducing the operator Δ or selecting only the proper connected diagram in perturbative expansions. By utilizing quite different

approach we could improve the calculation of the collision terms, which was done in most of paper [Eq. (3. 12) in (Ref. 22)] [Eqs. (5. 11) and (5. 12) in (Ref. 18)].

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