병렬시스템 가동률의 베이즈 추정량에 다하여

정 희 숙[1)], 金 章 郁[2)]

On The Bayes Estimation for the Availability of Parallel System

Hee-Sook Jung , Jang-Wook Kim

1. Introduction

We consider a maintained parallel system consisting of N nonidentical components/repair facilities, each of which has exponentially distributed failure and repair times. More precisely, for the i-th component, suppose that k_i failure/repair cycles are observed, $x_{i1}, x_{i2}, \cdots, x_{ik_i}$ are independent failure times distributed exponentially with the mean time between failures(MTBF) θ_i and $y_{i1}, y_{i2}, \cdots, y_{ik_i}$ are independent repair times distributed exponentially with the mean time beween repairs(MTBR) μ_i, then the i-th component availability is definded as

(1.1) $$A_i = \frac{\theta_i}{\theta_i + \mu_i}.$$

Since the parallel system unavailability $\bar{A}$ is the product of the component unavailabilities, $\bar{A} = \prod_{i=1}^{N} \bar{A}_i$, we obtain Bayes estimator of parallel system availability $A = 1 - \bar{A}$. The Bayes estimators are carried out under the noninformative and conjugate prior distributions.

By Sandler(1963), the parallel system unavailability $\bar{A}$ is the product of the

1)한국해양대학교 응용수학과 석사과정 응용통계 전공
2)한국해양대학교 응용수학과 교수

component unavailabilities;

(1.2) $$\overline{A}=\prod_{i=1}^{N}\overline{A}_i=\prod_{i=1}^{N}\left(\frac{\mu_i}{\theta_i+\mu_i}\right).$$

Thus, the parallel system availability becomes

(1.3) $$A=1-\overline{A}=1-\prod_{i=1}^{N}\left(\frac{\mu_i}{\theta_i+\mu_i}\right).$$

Sandler(1963) considered the system availability estimation for the two identical components in parallel system with one repair facility that the failure times for the two identical components are exponentially distributed with failure rate $\frac{1}{\theta}$ and the repair time for repair facility is exponentially distributed with repair rate $\frac{1}{\mu}$. Thompson and Springer(1972) carried out the Bayesian analysis of system availability on the N-component subsystem. Thompson and Palicio(1975) investigated a numerical procedure for computing Bayesian confidence intervals for the availability of a series or parallel system consisting of several independent two-state subsystems, whose failure and repair times are distributed exponentially.

The purposes of this thesis are to propose and study some Bayesian point estimators of the parallel system availability for a maintained parallel system consisting of N nonidentical components/repair facilities, each of which has exponentially distributed failure and repair times. We will also compare numerically the proposed Bayes estimates of the parallel system availability with the other estimates using Martz and Waller's data (1982).

2. Preliminaries

(1) The Prior Distributions :

Ever since the original scheme was proposed by Bayes, a crucial problem has been the prior distribution $g(\theta)$. How does one select a known prior distribution with density $g(\theta)$ to express the uncertainty about the unknown parameter θ? There may be some empirical evidence obtained through earlier experiments. It help us decide on the prior distribution $g(\theta)$. On the other hand, one can decide $g(\theta)$ or at least a class of prior $g(\theta)$ on a subjective basis or in a normative way. Various rules have been suggested and it appears that there is no best solution of the problem. The prior distribution $g(\theta)$ represents all about the parameter θ (either scalar or vector) with empirical data. Prior distribution may be categorized in different ways.

(2) Loss Functions :

Let $\hat{\theta}$ be a decision-maker's "best" guess of the unknown θ relative to a loss function $L(\hat{\theta}, \theta)$. Then the Bayes estimator is the guessed value which minimizes the posterior expected loss $E[L(\hat{\theta}, \theta)] = \int_{-\infty}^{\infty} L(\hat{\theta}, \theta)\pi(\theta \mid x_1, x_2, \cdots, x_n)d\theta$. In particular, if the loss function is the squared error $L(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2$, then, provided it exists, the Bayes estimator is the posterior mean.

(3) Bayes Estimators of θ :

The joint probability density function of $(X_1, X_2, \cdots, X_n, \theta)$ is

$$h(x_1, x_2, \cdots, x_n, \theta) = \prod_{i=1}^{n} f(x_i \mid \theta)g(\theta) = L(\theta; x_1, x_2, \cdots, x_n)g(\theta).$$

Bayes' Theorem tells us that the posterior density for θ given the data $(x_1, x_2, \cdots, x_n)$ is proportional to the product of the likelihood for θ given $(x_1, x_2, \cdots, x_n)$ and prior density for θ.

Since the marginal probability density function of $(X_1, X_2, \cdots, X_n)$ is

$$p(X_1, X_2, \cdots, X_n) = \int_{\Omega} h(x_1, x_2, \cdots, x_n, \theta)d\theta,$$

the conditional probability density function of θ given the data $(x_1, x_2, \cdots, x_n)$ is

$$\pi(\theta \mid x_1, x_2, \cdots, x_n) = \frac{h(x_1, x_2, \cdots, x_n, \theta)}{p(x_1, x_2, \cdots, x_n)} = \frac{L(\theta, x_1, x_2, \cdots, x_n)g(\theta)}{\int_{\Omega} L(\theta, x_1, x_2, \cdots, x_n)g(\theta)d\theta}.$$

Finding the posterior distribution becomes the Bayes estimator. When the squared error loss is used, the Bayes estimator of θ in the above set up is $\theta^* = E(\theta \mid x_1, x_2, \cdots, x_n) = \int_{\Omega} \theta\pi(\theta \mid x_1, x_2, \cdots, x_n)d\theta$.

3. Bayes estimators of the system availability

We assume that the failure times for the N nonidentical components are exponentially distributed with MTBF's $\theta_1, \theta_2, \cdots$ and θ_N ,respectively, such that

(3.1) $$h_1(x_i \mid \theta_i) = \frac{1}{\theta_i} \exp\left(-\frac{x_i}{\theta_i}\right),\ x_i > 0,\ \theta_i > 0,\ i = 1, \cdots, N,$$

and the repair times for the N nonidentical repair facilities are exponentially distributed with the MTBR's $\mu_1, \mu_2, \cdots$ and μ_N , respectively, such that

(3.2) $$h_2(y_i \mid \mu_i) = \frac{1}{\mu_i} \exp\left(-\frac{y_i}{\mu_i}\right),\ y_i > 0,\ \mu_i > 0,\ i = 1, \ldots, N.$$

For the i-th component, we obtain the likelihood function of T_{x_i} and T_{y_i}, for given θ_i and μ_i

(3.3) $$L_i(T_{x_i}, T_{y_i} \mid \theta_i, \mu_i) = \frac{1}{(\theta_i \mu_i)^{k_i}} \exp\left(-\frac{T_{x_i}}{\theta_i} - \frac{T_{y_i}}{\mu_i}\right),$$

where k_i is the observed failure/repair cycles, $T_{x_i} = \sum_{j=1}^{k_i} x_{ij}$ is total operating time, where x_{ij} is the j-th failure time, $T_{y_i} = \sum_{j=1}^{k_i} y_{ij}$ is total repair time, where T_{y_i} is the j-th repair time. Now, we introduce some integrations for useful calculations of the expectations, as follow,

(3.4) $$\int_0^\infty x^{a-1} \exp(-tz)\, dz = t^{-a} \Gamma(a) \quad \text{and}$$

(3.5) $$\int_0^\infty x^{b-1} \exp(1-x)^{c-b-1} (1-tx)^{-a} dx = \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)}\ {}_2F_1(a, b; c; t)$$

for $|t| < 1,\ c > b > 0$, where ${}_2F_1(a, b; c; t) = \sum_{i=0}^{\infty} \frac{(a)_i (b)_i}{(c)_i\, i!} t^i$ is a confluent hypergeometric function in Gauss' form for $|t| < 1$ with $(a)_i = \frac{\Gamma(a+i)}{\Gamma(a)}$ (Erdélyi,1953).

3.1. Under the noninformative prior distributions

We assume that the MTBF's θ_i and MTBR's μ_i have independent noninformative prior distributions, respectively, (3.1.1) $$f_{1i}(\theta_i) \propto \frac{1}{\theta_i^{u_i}}, \ \theta_i>0, \ u_i>0 \quad \text{and}$$

(3.1.2) $$f_{2i}(\mu_i) \propto \frac{1}{\mu_i^{v_i}}, \ \mu_i>0, \ v_i>0.$$

Then, we have the following lemmas and a theorem.

Lemma 3.1 The joint posterior distribution of θ_i and μ_i for the i-th component is

(3.1.3) $$\overline{f_{1i}}(\theta_i, \mu_i \mid T_{x_i}, T_{y_i}) = \frac{(T_{x_i})^{k_i+u_i-1}(T_{y_i})^{k_i+v_i-1}\exp\left[-\left(\frac{T_{x_i}}{\theta_i}+\frac{T_{y_i}}{\mu_i}\right)\right]}{\Gamma(k_i+u_i-1)\Gamma(k_i+v_i-1)\,\theta_i^{k_i+u_i}\mu_i^{k_i+v_i}}, \ \theta_i>0, \ \mu_i>0,$$

where $\Gamma(\cdot)$ is a Gamma function.

Proof. From (3.3),(3.1.1) and (3.12), the joint posterior distribution of θ_i and μ_i becomes

(3.1.4) $$\overline{f_{1i}}(\theta_i, \mu_i \mid T_{x_i}, T_{y_i}) = \frac{L_i(T_{x_i}, T_{y_i} \mid \theta_i, \mu_i) f_{1i}(\theta_i) f_{2i}(\mu_i)}{\int_0^\infty \int_0^\infty L_i(T_{x_i}, T_{y_i} \mid \theta_i, \mu_i) f_{1i}(\theta_i) f_{2i}(\mu_i) d\theta_i d\mu_i}.$$

The denominator of (3.1.4) reduces to

(3.1.5) $$\int_0^\infty \int_0^\infty \theta_i^{-(k_i+u_i)} \mu_i^{-(k_i+v_i)} \exp\left[-\left(\frac{T_{x_i}}{\theta_i}+\frac{T_{y_i}}{\mu_i}\right)\right] d\theta_i d\mu_i$$
$$=\left[\int_0^\infty \theta_i^{-(k_i+u_i)}\exp\left(-\frac{T_{x_i}}{\theta_i}\right)d\theta_i\right]\left[\int_0^\infty \mu_i^{-(k_i+v_i)}\exp\left(-\frac{T_{y_i}}{\mu_i}\right)d\mu_i\right].$$

By(3.4), $\int_0^\infty \theta_i^{-(k_i+u_i)}\exp\left(-\frac{T_{x_i}}{\theta_i}\right)d\theta_i = \frac{\Gamma(k_i+u_i-1)}{(T_{x_i})^{k_i+u_i-1}}$, $\int_0^\infty \mu_i^{-(k_i+v_i)}\exp\left(-\frac{T_{y_i}}{\mu_i}\right)d\mu_i = \frac{\Gamma(k_i+v_i-1)}{(T_{y_i})^{k_i+v_i-1}}$,

which completes the proof.

Lemma 3.2 The posterior distribution of the i-th component unavailability $\overline{A_i} = \frac{1}{1+\delta_i}$ is

(3.1.6) $$\overline{g_{1i}}(\overline{A_i} \mid T_{x_i}, T_{y_i}) = \frac{\left(\frac{T_{x_i}}{T_{y_i}}\right)^{k_i+u_i-1}(1-\overline{A_i})^{k_i+v_i-2}(\overline{A_i})^{k_i+u_i-2}}{B(k_i+u_i-1, k_i+v_i-1)\left[1-\overline{A_i}\left(1-\frac{T_{x_i}}{T_{y_i}}\right)\right]^{2k_i+u_i+v_i-2}},$$

where $0<\overline{A_i}<1$ and $B(\cdot,\cdot)$ is a Beta function and $\delta_i = \frac{\theta_i}{\mu_i}$ is the service factor.

Proof. First, we find the posterior distribution of $\delta_i = \frac{\theta_i}{\mu_i}$. To do this, let $\gamma_i = \mu_i$.

Then the joint posterior distribution of (δ_i, γ_i) is

$$\overline{f_{1i}}(\delta_i, \gamma_i \mid T_{x_i}, T_{y_i}) = \frac{(T_{x_i})^{k_i+u_i-1}(T_{y_i})^{k_i+v_i-1}(\gamma_i\delta_i)^{-(k_i+u_i)}\gamma_i^{-(k_i+v_i-1)}}{\Gamma(k_i+u_i-1)\Gamma(k_i+v_i-1)} \exp\left[-\left(\frac{T_{x_i}}{\gamma_i\delta_i}+\frac{T_{y_i}}{\gamma_i}\right)\right].$$

Hence, the marginal posterior distribution of δ_i is

$$\begin{aligned}\overline{g_{1\delta_i}}(\delta_i \mid T_{x_i}, T_{y_i}) &= \int_0^\infty \overline{f_{1i}}(\gamma_i\delta_i, \gamma_i \mid T_{x_i}, T_{y_i}) \mid J \mid d\gamma_i \\ &= \frac{(T_{x_i})^{k_i+u_i-1}(T_{y_i})^{k_i+v_i-1}\,\delta_i^{k_i+v_i-2}}{B(k_i+u_i-1, k_i+v_i-1)(T_{x_i}+\delta_i T_{y_i})^{2k_i+u_i+v_i-2}}.\end{aligned}$$

Therefore, the posterior distribution of the i-th component unavailability $\overline{A_i}$ is

$$\begin{aligned}\overline{g_{1i}}(\overline{A_i} \mid T_{x_i}, T_{y_i}) &= \overline{g_{1\delta_i}}(\overline{A_i}^{-1}-1 \mid T_{x_i}, T_{y_i})\,\overline{A_i}^{-2} \\ &= \frac{\left(\frac{T_{x_i}}{T_{y_i}}\right)^{k_i+u_i-1}(\overline{A_i})^{k_i+u_i-2}(1-\overline{A_i})^{k_i+v_i-2}}{B(k_i+u_i-1, k_i+v_i-1)\left[1-\overline{A_i}\left(1-\frac{T_{x_i}}{T_{y_i}}\right)\right]^{2k_i+u_i+v_i-2}}.\end{aligned}$$

Theorem 1 Under a squared error loss function, the Bayes point estimator of the parallel system availability A_i is

(3.1.7) $$A_{i1}^* = 1 - \prod_{i=1}^{N}\left(\frac{T_{y_i}}{T_{x_i}}\right)\left(\frac{k_i+u_i-1}{2k_i+u_i+v_i-2}\right) {}_2F_1\left(1, k_i+u_i; 2k_i+u_i+v_i-1; 1-\frac{T_{y_i}}{T_{x_i}}\right),$$

where $0 < \frac{T_{y_i}}{T_{x_i}} < 2$ and ${}_2F_1(a, b; c; t)$ is a confluent hypergeometric function in Gauss' form.

Proof. Since we use a squared error loss, we are only to find the posterior mean of the i-th component unavailability $\overline{A_i}$ as follows

(3.1.8)

$$\begin{aligned}E(\overline{A_i} \mid T_{x_i}, T_{y_i}) &= \int_0^1 \overline{A_i}\,\overline{g_{1i}}(\overline{A_i} \mid T_{x_i}, T_{y_i})\,d\overline{A_i} \\ &= \left(\frac{T_{y_i}}{T_{x_i}}\right)\left(\frac{k_i+u_i-1}{2k_i+u_i+v_i-2}\right) {}_2F_1\left(1, k_i+u_i; 2k_i+u_i+v_i-1; 1-\frac{T_{y_i}}{T_{x_i}}\right).\end{aligned}$$

Hance, the posterior mean of availability is

$$A_{i1}^* = 1 - \prod_{i=1}^{N}\left(\frac{T_{y_i}}{T_{x_i}}\right)\left(\frac{k_i+u_i-1}{2k_i+u_i+v_i-2}\right) \times {}_2F_1\left(1, k_i+u_i; 2k_i+u_i+v_i-1; 1-\frac{T_{y_i}}{T_{x_i}}\right).$$

3.2. Under the conjugate prior distributions

We assume that the MTBF's θ_i and MTBR's μ_i have independent conjugate prior distributions, respectively,

(3.2.1) $$f_{3i}(\theta_i) \propto \frac{b_i^{a_i}}{\Gamma(a_i)}\left(\frac{1}{\theta_i}\right)^{a_i+1}\exp\left(-\frac{b_i}{\theta_i}\right),\ \theta_i>0,\ a_i,\ b_i>0 \quad \text{and}$$

(3.2.2) $$f_{4i}(\mu_i) \propto \frac{d_i^{c_i}}{\Gamma(c_i)}\left(\frac{1}{\mu_i}\right)^{c_i+1}\exp\left(-\frac{d_i}{\mu_i}\right),\ \mu_i>0,\ c_i,\ d_i>0.$$

Then, we have the following lemmas and a theorem.

Lemma 3.3 The joint posterior distribution of θ_i and μ_i for the i-th component is

(3.2.3)

$$\overline{f_{2i}}(\theta_i,\mu_i \mid T_{x_i}, T_{y_i}) = \frac{(T_{x_i}+b_i)^{k_i+a_i}(T_{y_i}+d_i)^{k_i+c_i}}{\Gamma(k_i+a_i)\Gamma(k_i+c_i)\theta_i^{k_i+a_i+1}\mu_i^{k_i+c_i+1}}\exp\left[-\frac{1}{\theta_i}(T_{x_i}+b_i)-\frac{1}{\mu_i}(T_{y_i}+d_i)\right],$$

where $\theta_i>0,\ \mu_i>0$ and $\Gamma(\cdot)$ is a Gamma function.

Proof. The joint posterior distribution of θ_i and μ_i is

(3.2.4) $$\overline{f_{2i}}(\theta_i,\mu_i \mid T_{x_i}, T_{y_i}) = \frac{L_i(T_{x_i}, T_{y_i} \mid \theta_i,\mu_i)f_{3i}(\theta_i)f_{4i}(\mu_i)}{\int_0^\infty\int_0^\infty L_i(T_{x_i}, T_{y_i} \mid \theta_i,\mu_i)f_{3i}(\theta_i)f_{4i}(\mu_i)d\theta_i d\mu_i}.$$

The denominator of (3.2.4) becomes

(3.2.5) $$\int_0^\infty\int_0^\infty \theta_i^{-(k_i+a_i+1)}\mu_i^{-(k_i+c_i+1)}\exp\left[-\frac{1}{\theta_i}(T_{x_i}+b_i)-\frac{1}{\mu_i}(T_{y_i}+d_i)\right]d\theta_i d\mu_i$$
$$=\left[\int_0^\infty \theta_i^{-(k_i+a_i+1)}\exp\left\{-\frac{1}{\theta_i}(T_{x_i}+b_i)\right\}d\theta_i\right]\left[\int_0^\infty \mu_i^{-(k_i+c_i+1)}\exp\left\{-\frac{1}{\mu_i}(T_{y_i}+d_i)\right\}d\mu_i\right].$$

For i-th component, the joint posterior distribution of θ_i and μ_i is

$$\overline{f_{2i}}(\theta_i,\mu_i \mid T_{x_i}, T_{y_i}) = \frac{(T_{x_i}+b_i)^{k_i+a_i}(T_{y_i}+d_i)^{k_i+c_i}}{\Gamma(k_i+a_i)\Gamma(k_i+c_i)\theta_i^{k_i+a_i+1}\mu_i^{k_i+c_i+1}}\exp\left[-\frac{1}{\theta_i}(T_{x_i}+b_i)-\frac{1}{\mu_i}(T_{y_i}+d_i)\right].$$

Lemma 3.4 The posterior distribution of the i-th component unavailability $\overline{A_i}=\frac{1}{1+\delta_i}$ is

(3.2.6) $$\overline{g_{2i}}(\overline{A_i} \mid T_{x_i}, T_{y_i}) = \frac{\left(\frac{T_{x_i}+b_i}{T_{y_i}+d_i}\right)^{k_i+a_i}(1-\overline{A_i})^{k_i+c_i-1}(\overline{A_i})^{k_i+a_i-1}}{B(k_i+a_i,k_i+c_i)\left[1-\overline{A_i}\left(1-\frac{T_{x_i}+b_i}{T_{y_i}+d_i}\right)\right]^{2k_i+a_i+c_i}},$$

where $0 < \overline{A}_i < 1$ and $B(\cdot,\cdot)$ is a Beta function and $\delta_i = \frac{\theta_i}{\mu_i}$ is the service factor.

Proof. By the same manner of proof in Lemma 3.2, the posterior distribution of $\delta_i = \frac{\theta_i}{\mu_i}$ is

$$\overline{g_{2\delta_i}}(\delta_i \mid T_{x_i}, T_{y_i}) = \int_0^\infty \overline{f_{2i}}(\mu_i\delta_i, \mu_i \mid T_{x_i}, T_{y_i})\mu_i\, d\mu_i$$
$$= \frac{(T_{x_i}+b_i)^{k_i+a_i}(T_{y_i}+d_i)^{k_i+c_i}\delta_i^{k_i+c_i-1}}{B(k_i+a_i, k_i+c_i)[(T_{x_i}+b_i)+\delta_i(T_{y_i}+d_i)]^{2k_i+a_i+c_i}}.$$

Therefore, the posterior distribution of the i-th component unavailability $\overline{A}_i$ is

$$\overline{g_{2i}}(\overline{A}_i \mid T_{x_i}, T_{y_i}) = \frac{\left(\frac{T_{x_i}+b_i}{T_{y_i}+d_i}\right)^{k_i+a_i}(\overline{A}_i)^{k_i+a_i-1}(1-\overline{A}_i)^{k_i+c_i-1}}{B(k_i+a_i, k_i+c_i)\left[1-\overline{A}_i\left(1-\frac{T_{x_i}+b_i}{T_{y_i}+d_i}\right)\right]^{2k_i+a_i+c_i}}.$$

Theorem 2 Under a squared error loss function, the Bayes point estimator of the parallel system availability A_i is

(3.2.7) $$A_{l2}^* = 1 - \prod_{i=1}^{N}\left(\frac{T_{y_i}+d_i}{T_{x_i}+b_i}\right)\left(\frac{k_i+a_i}{2k_i+a_i+c_i}\right) {}_2F_1\left(1, k_i+a_i+1; 2k_i+a_i+c_i+1; 1-\frac{T_{y_i}+d_i}{T_{x_i}+b_i}\right),$$

where $0 < \frac{T_{y_i}+d_i}{T_{x_i}+b_i} < 2$ and ${}_2F_1(a,b;c;t)$ is a confluent hypergeometric function in Gauss' form.

Proof. By the same manner of proof in Theorem 1, the posterior mean of the i-th component unavailability $\overline{A}_i$ is

(3.2.8)

$$E(\overline{A}_i \mid T_{x_i}, T_{y_i}) = \int_0^1 \overline{A}_i\, \overline{g_{2i}}(\overline{A}_i \mid T_{x_i}, T_{y_i})\, d\overline{A}_i$$
$$= \left(\frac{T_{y_i}+d_i}{T_{x_i}+b_i}\right)\left(\frac{k_i+a_i}{2k_i+a_i+c_i}\right) {}_2F_1\left(1, k_i+a_i+1; 2k_i+a_i+c_i+1; 1-\frac{T_{y_i}+d_i}{T_{x_i}+b_i}\right).$$

Hence, the posterior mean of availability is

$$A_{l2}^* = 1 - \prod_{i=1}^{N} E(\overline{A}_i \mid T_{x_i}, T_{y_i})$$
$$= 1 - \prod_{i=1}^{N}\left(\frac{T_{y_i}+d_i}{T_{x_i}+b_i}\right)\left(\frac{k_i+a_i}{2k_i+a_i+c_i}\right) {}_2F_1\left(1, k_i+a_i+1; 2k_i+a_i+c_{i+1}; 1-\frac{T_{y_i}+d_i}{T_{x_i}+b_i}\right).$$

4. Examples

There is a repair facility for each component, and it is assumed that the repair rates are identical. Each component's availability is independent of the other. The following failure/repair data were obtained by

Component 1		Component 2	
Failure Times	Repair Times	Failure Times	Repair Times
74.3	0.5	128.3	11.8
19.0	10.1	17.8	4.8
26.7	5.8	47.8	3.6
88.5	1.2	5.2	5.0

(Martz and Waller's data, 1982)

Simulation used the mean time between failures (MTBF), $\theta_i = 60$ and the mean time between repairs (MTBR), $\mu_i = 4$. The true value of system availability(A_s) is equal to 0.996. From the data, $k_1 = 4$, $k_2 = 4$, $T_{x_1} = 208.5$, $T_{y_1} = 17.6$, $T_{x_2} = 199.1$ and $T_{y_2} = 25.2$. Thus, Bayes estumates of the parallel system availability are computed by Theorem 1 and Theorem 2. Consequently, under the noninformative priors, for $u_1 = 1$, $v_1 = 3$, $u_2 = 1$ and $v_2 = 3$, Bayes estimate of the parallel system availability is near the true value, and under the conjugate priors, for $a_1 = 1$, $b_1 = 2$, $c_1 = 2$, $d_1 = 1$ and $a_2 = 1$, $b_2 = 2$, $c_2 = 2$ and $d_2 = 1$, Bayes estimate of the parallel system availability is near the true value.

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(국문요약) **병렬시스템 가동률의 베이즈 추정에 대하여**

시스템의 가동률은 부품의 고장과 수리시간의 분포뿐만 아니라, 시스템의 외형과 수리시설의 개수와 같은 요인에도 좌우된다. 이 논문에서는 서로 동일하지 않는 N개의 부품들과 N개의 수리시설들로 구성되어 있는 지속적인 병렬시스템을 생각한다.여기서 각각의 부품들과 수리시설들의 고장시간과 수리시간은 모두 지수분포를 따른다고 가정한다. 이러한 상황하에서 병렬시스템 가동률의 베이즈 추정량(Bayes Estimator)을 몇가지 상황의 사전분포에 대해 연구하는 것이 이 논문의 궁극적인 목적이다.

예제에서는 이러한 제안된 베이즈 추정량을 기초로 Martz와 Waller(1982)의 생성 데이터를 이용하여 실지로 병렬시스템 가동률의 베이즈 추정치를 구하고 가동률의 참값(true value)과 비교해 다음과 같은 결과를 얻었다.

1) 비정보사전분포하에서는 사전분포의 모수 (u_1, v_1)과 (u_2, v_2)가 (1,3)과 (1,3)일 때, 추정치가 참값에 가장 가까운 값을 구할 수 있었다. 따라서 이 데이터에 대해서는 모수가 (1,3)와 (1,3)인 사전분포가 가장 적합하다고 생각된다.

2) 공액사전분포하에서는 사전분포의 모수 (a_1, b_1, c_1, d_1)과 (a_2, b_2, c_2, d_2)가 각각 (1,2,2,1)과 (1,2,2,1)일 때, 가장 참값에 가까운 추정치를 얻을 수 있었다.