論 文

Numerical Analysis of Ocean Wave by Multi-Grid Method

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복합격자 방법에 의한 해양파의 수치해석

곽 승 현

Key Words : Multi-Grid(복합격자), Free-Surface(자유표면), Viscous Flow(점성흐름), Triple
Mesh Method(삼중격자법), Navier-Stokes Equation(나비에스톡스 방정식)

Abstract

The ocean wave is hydrodynamically investigated to get more reliable solution. To improve the computational accuracy, more fine grids are used with relatively less computer storage on the free surface. One element of the free surface is discretized into more fine grids because the free-surface waves are much affected by the grid size in the finite difference scheme. Here the multi-grid method is applied to confirm the efficiency for the S103 ship model by solving the Navier-Stokes equation for the turbulent flows. According to the computational result, approximately 30% can be improved in the free surface generation. Finally, the limiting streamlines show that the numerical result is similar to the experiment by twin tuft.

1. Introduction

To overcome the deficiency of computer's hardware, the improvement of the efficiency has been strongly demanded in the computational procedure. Many numerical techniques have been developed to improve the finite difference method, but the method still faces a serious problem because it requires very long CPU time and a huge memory storage for accurate simulation. The method of IAF (implicit approximation factorization), a kind of implicit scheme, the method of local time step, etc. are the examples for more efficient computations. Some comparative calculations by these methods have been carried out. It seems that IAF is

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quite promising to speed up the calculation but its formulation is a little complicated. Moreover, the method brings forth some numerical damping. For the numerical truncation error to be small enough to have little effect on the physical performance, the mesh size should be strictly considered because it is very crucial for the efficiency. For example, in the finite difference calculations of viscous flow described by the Navier-Stokes equation, the minimum mesh size is usually so chosen that the numerical dissipation, which comes from the discretization of the convection terms, is much less than the physical dissipation. The mesh size must be extremely small for high Reynolds-number flows to meet this demand. However, such fine meshes are not always necessary for all the equations and terms. For example, the truncation errors of the Poisson equation for the pressure of the non- convective terms in Navier-Stokes equation do not have much influence on the results as the convection terms do. The hybrid type of the mesh may make the computations more efficent. One possibility is to employ different mesh systems depending on the characteristics of the equations or the terms. We call such a method "double mesh method[1]" or "triple mesh method[2]", written in short as DMM or TMM hereafter. It was first proposed for numerical simulations of 3-D nonlinear free-surface flow problems by boundary element method[3]. In order to reduce the numerical viscosity as much as possible, a very fine mesh system which contains about 60 grids[4] in one wave length is used in the finite difference calculation concerned with the free-surface equations, while the governing Laplace equation is solved on a relatively coarse

mesh system which contains about 10 grids in one wave length by the boundary element method. The computed results by DMM or TMM were of enough accuracy and both the computing time and the size of the memory storage were remarkably reduced.

In the present study, a multi-grid on the free-surface is introduced in the finite difference solver of the Navier-Stokes equation to improve the calculation efficiency. As mentioned above, the demands to the mesh size are not the same for all the equations and the terms in the finite difference method. So it is expected that some improvement, similar to that achieved in the simulation of free-surface problem by DMM or TMM, may be made by introducing more fine meshes in the conventional finite difference method.

2. Governing Equations and Computational Strategy

A single grid system[5] is usually used in the whole computation whose minimum size is determined for the numerical diffusion to be less than that by viscosity. The grid size for the calculation of the free surface elevation must be determined by a different scale, the minimum wave length. In the simulation, two or three mesh systems are usually used whose sizes are different each other depending on the char- acteristic of equations. The first one is for the convective terms in the Navier-Stokes equation, the second is for the Poisson equation, and the third is for the free-surface equation. The third grid system requires the finest mesh. In the present calculation, the third one is numerically

confirmed ; more fine grids are used to improve the accuracy of free-surface calculation with relatively less computer storage. One element of the free-surface is discretized into (4xii,4xjj), (8xii,4xjj), (12xii, 4xjj) fine grids because the free-surface waves are much affected by the grid size in the finite-difference scheme.

The positions, or Lagrangian coordinates, of each particle (x^n_p, y^n_p, z^n_p) are obtained by numerical integration from some initial position (x^0_p, y^0_p, z^0_p) at time = 0 ;

$$x^n_p = x^0_p + \int u_p \cdot dt$$
$$y^n_p = y^0_p + \int v_p \cdot dt \qquad (1)$$
$$z^n_p = z^0_p + \int w_p \cdot dt$$

where u_p, v_p, w_p are the velocities in the Eulerian mesh at the time dependent location of the particle. In the present MAC-based codes, the particle velocities are evaluated by two-variable linear interpolation. Consistent with the forward time integration of MAC method, (1) is evaluated sequentially as (2).

$$x^{n+1}_i = x^n_i + u^n_i \cdot \triangle t$$
$$y^{n+1}_i = y^n_i + v^n_i \cdot \triangle t \qquad (2)$$
$$z^{n+1}_i = z^n_i + w^n_i \cdot \triangle t$$

(2) is the Lagrangean expression of the kinematic condition on the free-surface. The condition can also be expressed in the Euler form as follows;

$$\partial \zeta / \partial t = - u \cdot \partial \zeta / \partial x - v \cdot \partial \zeta / \partial y + w \qquad (3)$$

where ζ and t are the free-surface elevation and the time respectively. Numerically (2) is equiv-

alent to (3) if the 1st order upstream difference scheme is used in (3).

The shape of the free-surface is not known a priori; it is defined by the position of the marker particles. We note that the boundary conditions at the free-surface require zero tangential stress and a normal stress which balances any externally applied normal stress. The application of these conditions requires a knowledge of not only the location of the free-surface at each grid but also its slope and curvature. In our calculation, the z-coordinate of the free surface is re-arranged by the bivariate linear interpolation in proportion to the newly calculated projected area at each time step. At the hull surface, the no-slip condition is used for the velocity, and the Neumann condition for the pressure. The hull is fixed in the computational domain, while the uniform flow is imposed on the upstream boundary.

3. Application and Discussion

The S-103 model for high Reynolds-number flows is studied to confirm the numerical efficiency of the multi grid. S-103 is an Inuid model with the beam/length ratio of 0.09. In the present case, calculations are made at $R_n = 10^6$ and $F_n = 0.28$ with Baldwin Lomax turbulence model. The result is that at the time T=3.0, when the convergence is well assured. The grid size of regular type is 74x29x30 and the multi-grid on the free-surface is numerically tried. Fig. 1 shows the wave patterns obtained by the regular grid. Fig. 2 uses the grid of (4xii,4xjj) on free surface and gives us about 7% improvement in the free-surface development, compared with that
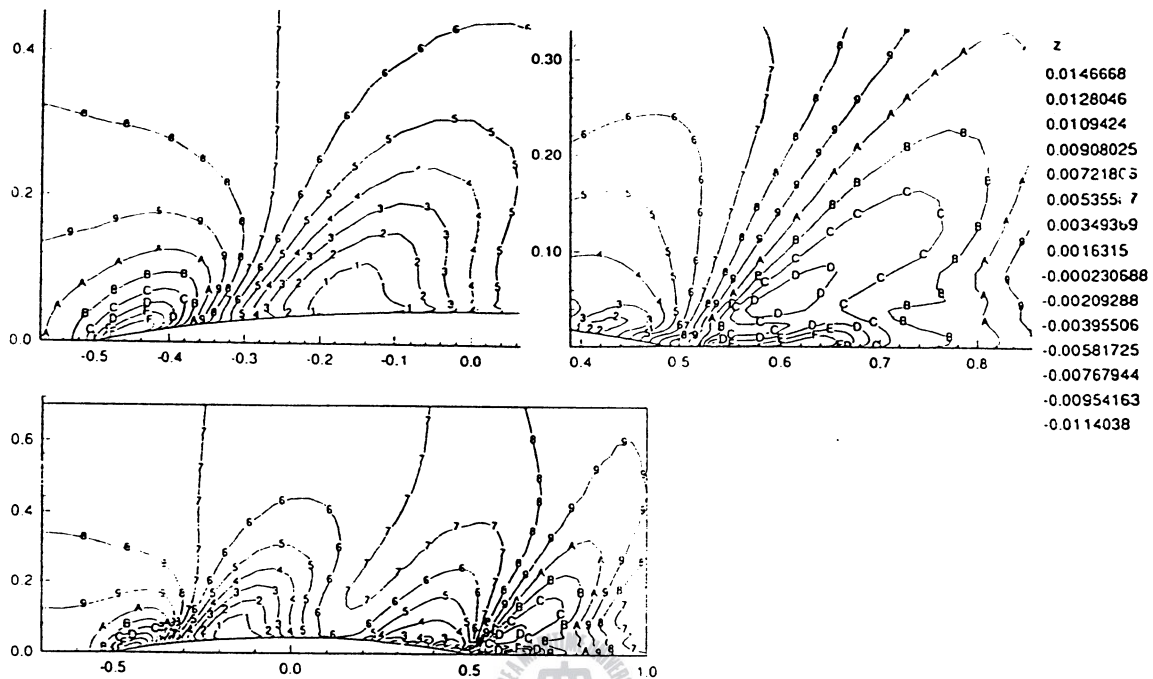
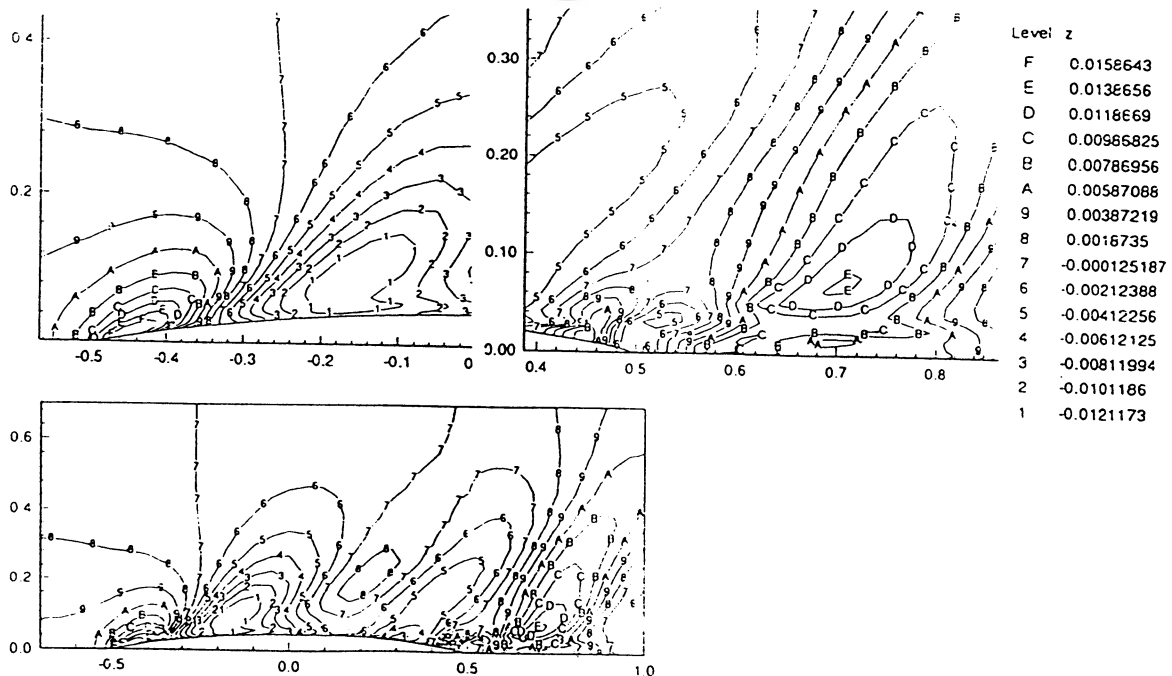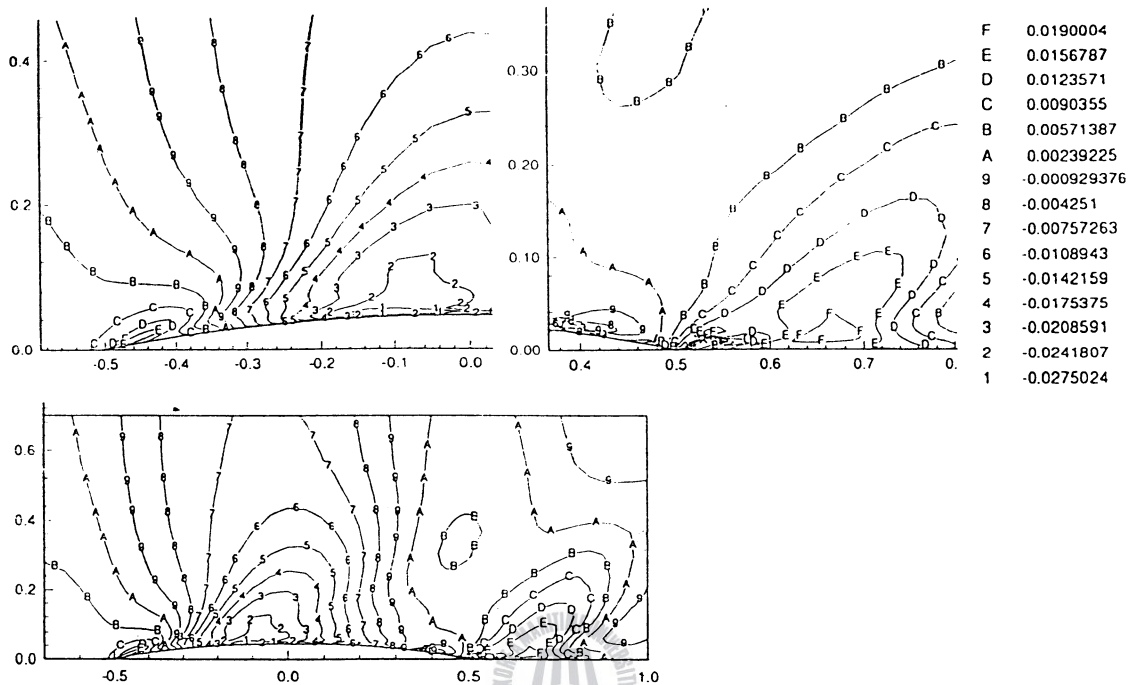Fig. 1   Free-surface contour by regular grid(ii, jj) for S-103 case

Fig. 2   Free-surface contour by multi-grid (4xill, 4xjj) for S-103 case

F	0.0190004
E	0.0156787
D	0.0123571
C	0.0090355
B	0.00571387
A	0.00239225
9	-0.000929376
8	-0.004251
7	-0.00757263
6	-0.0108943
5	-0.0142159
4	-0.0175375
3	-0.0208591
2	-0.0241807
1	-0.0275024

Fig. 3   Free-surface contour by multi-grid (8xill, 4xjj) for S-103 case

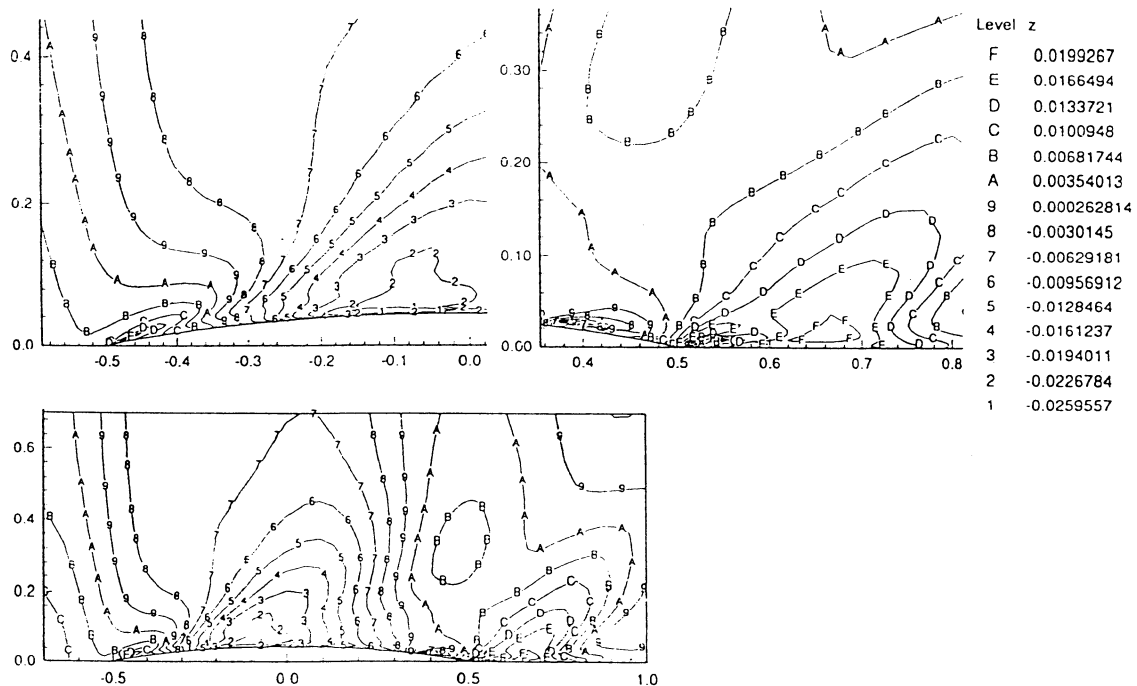| Level z	
F	0.0199267
E	0.0166494
D	0.0133721
C	0.0100948
B	0.00681744
A	0.00354013
9	0.000262814
8	-0.0030145
7	-0.00629181
6	-0.00956912
5	-0.0128464
4	-0.0161237
3	-0.0194011
2	-0.0226784
1	-0.0259557

Fig. 4   Free-surface contour by multi-grid (12xill, 4xjj) for S-103 case