A Study on the Origin of Plastic Spin

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요　　약

지금까지 소성스핀은 내부응력(역응력)에 의한 재료의 이방성에서 유래한다고 알려져 왔다. 소성스핀은 역응력의 객관성률에 적용되어 왔고, 또한 Jaumann률을 사용할 때 단순전단에서 해가 요동하는데 이에 대한 한 해결책으로도 사용되어 왔다. 그러나 소성스핀의 발생 원인에 기본적인 잘못이 있다고 사료된다.

본 연구에서는 결정재료의 소성변형을 고찰하고 종래와는 다른 소성스핀 발생기구를 제시한다. 소성변형에 대한 재료의 기하학적인 특성과 2가지 이상의 응력성분이 동시에 작용해야 소성스핀은 발생한다. 그러므로 종래에 알려진 바와 달리 소성스핀은 특별한 경우를 제외하고는 미미하고, 초기의 재료직조(texture)에 민감하다.

Abstract

It has been proposed that plastic spin originates from material anisotropy such as internal

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stress. The plastic spin has been applied to the objective rates of back stresses and used as the cure for the oscillation of solution in pure shear with Jaumann rate. However, there have been likely to be some fundamental misleadings in the origin of plastic spin.

In this paper, plastic deformations of polycrstalline materials are discussed in detail, and new origins for plastic spin are suggested in which it originates from geometric properties of plastic deformation and from the simultaneous stress components. It is also shown that the plastic spin is not significant and is sensitive to initially given texture.

Ⅰ. Introduction

Recently many researches have been performed on the plastic spin. Most of them have inclined to provide additional constitutive equations for plastic spin since Kratochvil's statement [1]. There are two typical approaches according to the origins of plastic spin: isotropic tensor representation [2] and generalized plastic potential one [3,4]. They have been attemped to make a connection between plastic spin and material anisotropy from internal stress because it has been believed that the spin originates from such anisotropy [5].

However, there have been likely to be some fundamental misleadings in the origin of plastic spin. The plastic spin originates not from the material anisotropy such as internal stress but from the geometric properties of plastic deformations and the simultaneous stress components.

In this paper, some aspects of basic concept for the plastic deformations of polycrstalline materials are discussed in detail, such as normality rule and internal stress which are related to the mechanism of plastic spin. And new origins for plastic spin are suggested. The plastic spin appears not significant but very sensitive to initially given texture through some simple calculations. The first tendency is contrast to the published results so far. It is also noticed that a macroscopic internal stress tensor can not represent internal stress status because such tensor is not defined in good approximation.

Ⅱ. Definition of Plastic Deformation Velocity Gradient

Consider a material point in polycrystal, whose dimension is sufficiently large for the representative size of grains so that its macromechanical properties are stationary for the sampling point, which is still regarded as a point in the sense of macroscopic treatment. Assuming uniform distributions in the stress and deformation of a grain, the macroscopic strain and stress are defined as a volume weighted average over grains. The same context holds for tensor inner product between stress and strain, and is also valid for its rates provided that the effects of rotation and convection can be neglected [6].

$$\sigma : \dot{e} = [\sigma_g : \dot{e}_g] \tag{2.1}$$

where subscript g means grain, [·] volume average over grains.

Many intergranular accommodation models can be obtained, such as Talyor's, Lin's, Kroner's and static ones, by giving specific accommodation function [7,8]. It is, therefore, plausible for the practical application to assume the static model. By this assumption and exclusion of elastic strain, Eq. (2.1) can be recast to

$$\sigma : \dot{e}^p = [\sigma : \dot{e}^p_g] \tag{2.2}$$

In general, the macroscopic plastic strain rate can not be obtained straightforwardly as Eq. (2.2) [6,9].

On the other hand, observe the plastic deformation phenomena of crystalline meterials in the macroscopic level. The macroscopic plastic deformation velocity gradient can be described by summing up the finite number of deformation velocity gradient corresponding to each slip system.

$$L_p = \frac{1}{n_g}\left(\sum_{j=1}^{ng}\sum_{j=1}^{ns}\dot{\gamma}_{ij}P_{ij}\right) \tag{2.3}$$

where $\dot{\gamma}_{ij}$ is the slip rate of system (ij), P_{ij} the plastic deformation velocity gradient for unit slip rate, and all grains are assumed to have the same volume. Here it is worth noting that the terms in parenthesis is not symmetric in general, thus plastic spin is defined antisymmetric part of the gradient.

Ⅲ. Aspect of Normality Rule

A macroscopic yield surface of crystalline material consists of many constraints by which all of the crystal stresses lie within the surface [6]. Under the assumption of Chapter 2, the yield surface of a polycrystalline appears just the envelope of the yield hyperplanes corresponding to each slip system in stress component space. Hereafter the normality of the flow rule will be discussed apart from the stable material postulate [10] or the maximum plastic dissipation postulate [11].

Taking a slip system in Eq. (2.3), whose normal vector is n_1 and slip direction s_1, the plastic deformation velocity gradient due to the slip is given to

$$L_l^p = \dot{\gamma}_1 P_1^s \tag{3.1}$$

where $P_1^s (= s_1 \otimes n_1)$ is a deviatoric tensor, $\dot{\gamma}_1$ a slip rate. Assuming Schmid law holds, it can be supposed that the yielding occurs when the Schmid stress τ_1^s of the slip system reaches at a certain critical stress τ^{cr}. Schmid stress τ_1^s is given as

$$\tau_1^s=\sigma : P_1^s \tag{3.2}$$

Taking the gradient of the hyperplane with respect to stress and comparing its components with those of the deviatoric tensor, we have the normality relation between the yield surface and plastic deformation velocity gradient.

Now we investigate the normality at vertex for polycrystals. Take another slip system noted by ②, and suppose the yield plane of system ② meets that of system ① at point Q. Consider a stress tensor at the point, by which two slips are ready to act.

$$\sigma : P_1^s=\tau_i^{cr},\ i=1, 2 \tag{3.3}$$

To define a unique flow direction, let's take mean surface and resultant plastic deformation velocity gradient by arithmetically averaging and adding respectively.

$$0.5\sigma : (P_1^s+P_2^s)-0.5(\tau_1^{cr}+\tau_2^{cr})=0 \tag{3.4}$$

$$L^p=\dot{\gamma}_1(P_1^s+P_2^s) \tag{3.5}$$

From the gradient of Eq. (3.5), there is a normality. It is, consequently, inferred that the requirement of the same slip rates for the slips simultaneously active at vertex is crucial for the normality with mean surface.

Previous discussions have been done without back stress. The normality holds for every where except vertex even in the case of back stress. Another normality relation will be defined at vertex. That is with average yield surface weighted by their slip rates.

$$\sigma : \frac{1}{\dot{\gamma}_1+\dot{\gamma}_2}(\dot{\gamma}_1 P_1^s+\dot{\gamma}_2 P_2^s)-\frac{1}{\dot{\gamma}_1+\dot{\gamma}_2}(\dot{\gamma}_1\tau_1^{cr}+\dot{\gamma}_2\tau_2^{cr})=0 \tag{3.6}$$

$$L^p=\dot{\gamma}_1 P_1^s+\dot{\gamma}_2 P_2^s \tag{3.7}$$

The normality is a universal property of crystalline materials as far as Schmid law holds, and the plastic deformations are confined to slips since the normality comes from the geometric nature of plastic deformation mechanism. So it is valid regardless of softening, hardening, rate independent or depentent plasticity. But an attention must be given to the normality at vertex in the case of finite back stress.

The normality has been discussed with slips. There is an energy concept behind the mechanism of normality previously discussed. It can be supposed a yielding occurs when a specific elastic energy given by unit strain in the direction of deviatoric tensor reaches a certain criteria. A yield function is, therefore, an energy criteria. By this context the normality holds universally for the crystalline materials under the assumption of incompressible plasticity and Schmid law.

Ⅳ. Aspect of Plastic Spin

In recent years much progress has been made in the plastic spin. Unfortunately there have been some misleadings, which had come from misinterpretation for the fundamental mechanism of the spin. It has been believed that the plastic spin originates from anisotropy such as internal stress due to substructure. Therefore, a connection between internal stress and plastic spin has been attempted in many studies, which leads a source of misleading. A new mechanism for plastic spin will be suggested in this chapter.

As mentioned before in Chapter 2, the plastic spin is a direct result of the slip. It is useful to examine some experiments with and without internal stress. An important consequence comes forth such that the plastic spin can bring about regardless internal stress. To sustain this consequence, Boukadia and Sidoroff [12] can be introduced who showed the plastic spin for a perfect plastic F.C.C. single crystal.

The fundamental mechanisms for plastic spin are suggested : 1) the non - homogeneity of plastic deformation mechanism(finite discrete slip), 2) texture, 3) stress state in the axis texture(for macroscopic level spin) or slip arrangement axis of a grain(for grain level spin), such that more than one component should be simultaneously applied, whose axial stress components have different magnitude. The third mechanism is responsible for the different Schimid stress induced on slip pairs. In a word, plastic spin origin from geometric nature of plastic deformation is associated with stress state.

With new interpretations, some results in literatures will be discussed in the next isotropic function representation for plastic spin [2,5,13,14,15]. The most simple one is

$$W^p = \eta(\alpha D^p - D^p \alpha), \tag{4.1}$$

where η is a scalar for fitting α internal stress. It is helpful to give an attention to

$$W_s^p = (s \otimes s) D_s^p - D_s^p (s \otimes s), \tag{4.2}$$

where W_s^p is a plastic spin and D_s^p, the symmetric part of plastic deformation veloctiy gradient tensor for single slip, which are expressed as follows, respectively.

$$W_s^p = \frac{1}{2}\dot{\gamma}(s\otimes n - n\otimes s) \tag{4.2a}$$

$$D_s^p = \frac{1}{2}\dot{\gamma}(s\otimes n + n\otimes s) \tag{4.2b}$$

The resemblance of Eqs. (4.1) and (4.2) was noticed by Giessen[16]. It shoud not be overlooked the importance of Eq. (4.2) since this was derived from kinematics, not from contitutive equation as Eq. (4.1). The proposed machanism for the plastic spin in this paper is compatible with Eq. (4.2). Isotropic function representation is very plausible, however, there has been misleadings in the application.

There is another approach in which unsymmertric internal stresses play an important roll[3,4]. PIOS(Plastic Induced Orientation Structure), whose reference state is specified by triad directors, is introduced in the theory. It is thought that PIOS is incorporated to texturing, orientation and intensity. If PIOS implies texturing, two variables are outstanding: intensity and orientation. Their conjugate forces may be given as follows,

$M^0 = \Sigma M_i$; for orientation

$M^1 = \Sigma \| M_i \|$; for intensity

where $\| \cdot \|$ is a norm.

A difficulty is shown which is concerned with the representation of macroscopic internal variable for internal stress in stress. Consider a slip system, and suppose that τ^α is an internal stress of the slip, τ^s Schmid stress, σ_i Cauchy stress. Then σ_i^α unknown component of internal stress(unknown also whether or not symmetric) can be found as follows and its existence is also proved,

$$\sigma_i^\alpha = (1-\gamma)\sigma_i, \tag{4.3}$$

where $\lambda = (\tau^s - \tau^\alpha)/\tau^s$ is a scalar. From Eq. (4.3) internal stress is a symmetric tensor because so is Cauchy stress. If a macroscopic internal stress is symmetric, Schmid stress can be written as follows.

$$\sigma_{ij}[P_{ij}^s]_{sym} = \tau^\alpha, \tag{4.4}$$

where six equations are needed to determine the stress components uniquely. If the stress is anti-symmetric tensor, three equations are needed. Nine equations are necessary in the case that the combination of symmetric and anti-symmetric stress tensor also provides a solution. Thus there rarely exits unique solution in a crystal.

Directly speaking, a macroscopic internal stress in six or nine component stress space can not be used as internal variable in the finite plastic deformation of crystalline materials. In this context, it is expected much errors in the conventional approach. Really, we have had a evidence, such as severe distortions in yield surface [17,18]. The distortions can not be explained by one macroscopic internal stress tensor. The sharpening and flattening of yield surface around loading point or other distortions will be explained by the multi – internal stress tensor.

It is elucidated in the prvious discussions that the plastic spin arises regradless of the internal stress, a macroscopic internal stress can not be an internal variable in the case of finite plastic deformation, and the plastic spin dissipates no energy without couple stresses. Finally consider whether or not the plastic spin is subjected to thermodynamic law. Kratochvil [1] is the first that pointed out the plastic spin must be given by constitutive relation. From another view point, however, the plastic spin is one given by a constraint relation. When a system is symmetrized, the plastic spin hands over its restrictions in the thermodynamics to the symmetric variable and keeps only some relation with the symmetric variable. Thus the plastic spin is said to be subjected to thermodynamic law indirectly. The restriction to the symmetric one is just the restriction to the plastic spin. For instance,

$$\sigma : D^p = \sigma : f^{-1}(W^p) \geq 0 \tag{4.5}$$

V. Numerical Experiment for Plastic Spin

To validate the proposed origin for the plastic spin simple numerical calcuiations are performed based on Eq. (2.3) with a few grains.

1. Kinematics

Although kinematics is a fundamental theory in mechanics, it is still an open field especially in elastoplastic problems. A number of formulations for finite deformations have, as well known, been proposed, but no one has been given to a unified agreement yet. On one hand, many numerical schemes for solving boundary value problems employ updated Lagrangian method. Fortunately, the relative discrepancies from the different formulations may be trivial with updated Lagrangian one whose increment is small. The formulations in this paper aim to apply to the updated Lagrangian method.

Multiplicative decomposition of deformation gradient tensor has been used for elastoplastic deformations. Deformation velocity gradient is introduced as follows which is quite frequently seen in the literatures.

$$L=\dot{F}^{e}(F^{e})^{-1}+F^{e}\dot{F}^{p}(F^{p})^{-1}(F^{e})^{-1}=L^{e}+F^{e}L^{p}(F^{e})^{-1} \tag{5.1}$$

where the second term is not only odd but also difficult to measure in both experiments and numerical calculations. Thus a different approach shall be pursued in what follows.

The new approach is based on Lagrangian measures. The increment of Green strain tensor can be written as follows with multiplicative decomposition.

$$\Delta E=\frac{1}{2}(F^{T}F-1^{u})=\frac{1}{2}\{(F^{p})^{T}(F^{e})^{T}F^{e}F^{p}-(F^{p})^{T}F^{p}+(F^{p})^{T}F^{p}-1^{u}\}=(F^{p})^{T}\Delta E^{e}F^{p}+\Delta E^{p} \tag{5.2}$$

where 1^{u} is deagonal unit tensor, ΔE^{e} is elastic strain increment measured from x^{o}(initial reference configuration) and ΔE^{p} plastic strain increment from x^{p}(configuration after plastic deformation).

Now trying to evaluate plastic deformation gradient by approximation. By integration the relation

$$L^{p}=\dot{F}^{p}(F^{p})^{-1} \tag{5.3}$$

with an assumption of constant L^{p} in the course of increment, we have

$$F^{p}=E^{(L^{p}\Delta t)} \tag{5.4}$$

where Δt is a small time increment. Equation (5.4) can be evaluated by characteristic equation theory for a function of matrix. To avoid eigenvalue analysis, Eq. (5.4) is approximated as follows.

$$F^{p}=1^{u}+L^{p}\Delta t \tag{5.5}$$

2. Objective rate and stress increment

The time derivative of Lagrangian tensor has Lagrangian objectivity. It is convenient for deriving an objective rate to make use of Lagrangian tensor. Note the relation between Cauchy stress and the second Piola - Kirchoff stress

$$\sigma=\frac{1}{J}F\sigma_{p}F^{T} \tag{5.6}$$

where σ is Cauchy stress, σ_{p} the second Piola - Kirchoff stress on a certain configuration, J Jacobian of deformation gradient F. By taking time derivative of both sides in Eq. (5.6) and rearranging it, the objective rate which will be used in this paper is defined as

$$\hat{\sigma}^{T}=\frac{1}{J}F\dot{\sigma}_{p}F^{T}=\dot{\sigma}-L\sigma-\sigma L^{T}-\mathrm{tr}(L)\sigma \tag{5.7}$$

where $\overset{\circ}{\sigma}^T$ is Truesdel stress rate.

The constitutive relation between the second Piola - Kirchoff stress and Green strain tensor may be given in the incremental form as below.

$$\Delta\sigma_p = C\Delta E^e = C\{(F^p)^T(\Delta E - \Delta E^p)(F^p)^{-1}\} = \overline{C}(\Delta E - E^p) \quad (5.8)$$

where C is a forth order tensor and $\overline{C} = (F^p)^T C (F^p)^{-1}$. The update of stress is performed as follows.

$$\Delta\sigma = \frac{1}{J^e} F^e \Delta\sigma_p (F^e)^T + L\dot{\sigma}\Delta t + L\dot{\sigma}\Delta t + \mathrm{tr}(L)\dot{\sigma}\Delta t \quad (5.9)$$

where $\dot{\sigma} = \frac{1}{J} F\sigma^o F^T$, σ^o is Cauchy stress in x^o configuration before updating.

3. Slip velocity and hardening models

Many models are available for the slip velocity. In this study a viscoplastic model is considered. Gilman [19] suggested a slip model as follows.

$$\dot{v} = 2bv_o(\rho_o + \rho v)e^{-\frac{H_o + H_1 v}{\tau}} \quad (5.10)$$

where b is burger's vector, v_o the limit velocity of dislocation, H_o material constant, ρ_o initial dislocation density, ρ dislocation increasing rate to slip, τ shear stress. H_1 is concerned with representation of fixed portion of the generated dislocations, which actually means hardening. This velocity model is the base for the present examples, but slightly sophisticated modifications are given to it.

The hardening due to the fixed dislocation in the way of defects is modelled by the next evolution equation.

$$\dot{H}_s = c_s(h_s - H_s) \, \| \dot{v} \| \quad (5.11)$$

where c_s is material constant, h_s the saturation value of the hardening, and $\| \dot{v} \|$ a norm evaluated over all active deviatoric tensor. The hardening due to the fixed dislocations is assumed linearly increasing with slip distance.

$$H_1 = h_1 \, | \, v_m \, | \quad (5.12)$$

where h_1 is material constant, v_m maximum slip. The internal stress α is described by the evolution equation as follows.

$$\dot{\alpha} = c_b(h_b\dot{v} - \alpha\,\dot{v})$$
$$\dot{\alpha} = c_b(h_b\dot{v} - \alpha\,\dot{v}) \quad (5.13)$$

where c_b is material constant, h_b the saturation of the internal stress.

Finally the stress parameter is modelled to depend on the power (n). Gathering the above models, the slip velocity is modified as below.

$$\dot{\upsilon} = 2b\upsilon_o(\rho_o + \rho\upsilon_m)e^{-\frac{(H_o)^n + H_s + H_1(\tau^s - \alpha)^{n-1}}{(\tau^s - \alpha)^n}} \tag{5.14}$$

4. Numerical experiment

A plane stress problem is attempted for numerical experiment of the plastic spin. The loading is imposed by the prescribed total deformation tensor.

$$F_{11} = 1 + \delta\varepsilon_1,\ F_{12} = 0.0,\ F_{21} = \delta\varepsilon_2,\ F_{22} = 1.0$$

The (0, 0, 1) orientation of crystal is alligned to Z – axis.

Figures 5.1~5.3 show the plastic spin for shear loading. The considered orientation patterns are given in Table 5.1. As seen in figures, there is no significant plastic spin over the orientation patterns of grain. That implys that simultaneously applied stress components are essential for the plastic spin.

Table 5.1. Orientation patterns (unit deg)

P1	P2	P3
0.0	0.0	0.0
30.0	22.5	22.5
60.0	67.5	45.0
120.0	112.5	112.5
150.0	157.5	135.0
210.0	202.5	202.5
240.0	247.5	225.0
300.0	292.5	292.5
330.0	337.5	315.0

Figures 5.4~5.5 are the results of combined loading of shear and tension. It can be seen that the plastic spin is significant in Figure 5.5 which is for initially given texture contrary to Figure 5.4 for the even distribution of orienta-

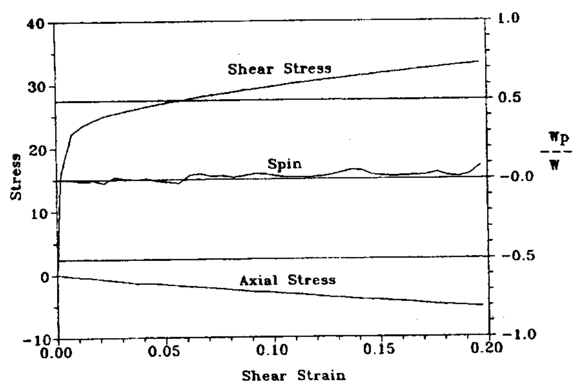

Fig. 5.1. Plastic spin for shear loading with orientation pattern 1

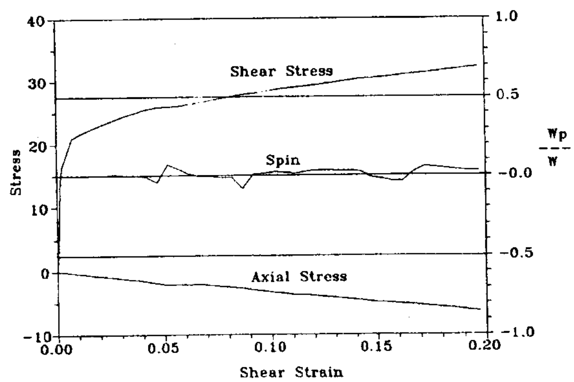

Fig. 5.2. Plastic spin for shear loading with orientation pattern 2

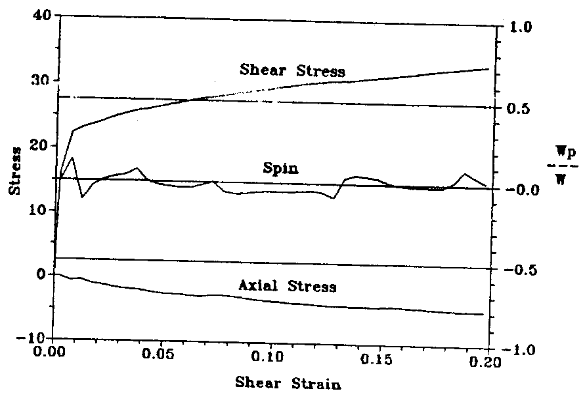

Fig. 5.3. Plastic spin for shear loading with orientation pattern 3

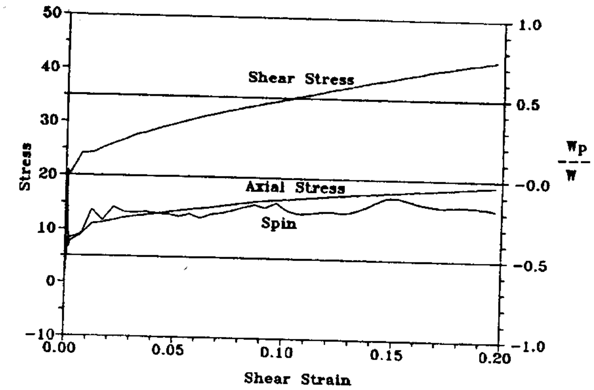

Fig. 5.4. Plastic spin for combined loading with even distribution of orientation pattern

tion. These figures show that the pattern of texture is also essential for plastic spin.

Generally speaking, the plastic spin is small and there is no need to consider it in the engineering problems except for the special case of initially textured material. The oscillations and spins in the figures are due to the small number of grains considered. If sufficient grains are considered, better results could be obtained.

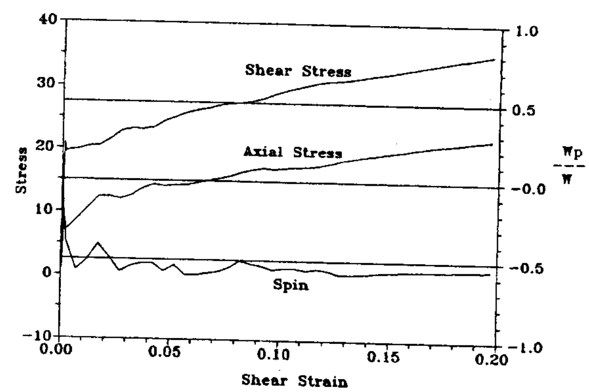

Fig. 5.5. Plastic spin for combined loading with initially given texture pattern

Ⅵ. CONCLUSION

In this paper, the plastic deformations of polycrstalline materials were discussed in detail and new origins for the plastic spin were suggested, from which the following conclusions are obtained:

1) The plastic spin originates from geometric properties of the plastic deformations.
2) For the plastic spin in polycrstalline materials, texture and simultaneously appliedstress components are essential.
3) The plastic spin is not significant except the especial cases differently before.
4) The anisotropy of internal stress has the secondary effects on the plastic spin.
5) The status of internal stress (eg. back stress) of crystalline materials can not be respresented by single macroscopic tensor.

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